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Electrical Conductivity of the Great Lakes

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A critical examination has been made of the electrical conductivity values assigned, for radio propagation purposes, to the waters of the Great Lakes. Discrepancies between conductivity values measured in the laboratory and those deduced from field strength measurements are shown to have been the result of both experimental error and the use of faulty standard theoretical field strength curves. The latter source of error is the result of error in the standard curves themselves and in the use of a dielectric constant of 15 for over-water propagation. Conductivity values derived from laboratory measurements of water samples are significantly different from those published in conductivity maps. Large seasonal variations in conductivity, approaching a factor of two in some cases, are a significant complicating factor.

1. Introduction

This investigation arose out of a discrepancy in conductivity values assigned to the Great Lakes: on the one hand, from field strength measurements and, on the other, from laboratory measurements on water samples.

The Telecommunications Branch of Canada's Department of Transport, which is responsible for radio licensing in Canada, desired to establish more accurate values for the conductivity of the Great Lakes. To this end they approached the National Research Council for the use of NRC's Motor Vessel *Radel II* in conducting field strength measurements on the four Great Lakes which border on Canada. At that time the author of this paper requested that water samples and temperature soundings be taken at a number of points in each of the lakes during the course of the field strength measurements. During the summer of 1959 these measurements were made and the water samples taken.

From the results of these trials a member of the technical staff of the Department of Transport deduced the conductivity of Lakes Superior, Huron, Erie, and Ontario by fitting experimental field strength versus distance curves to a set of theoretical curves published by the Federal Communications Commission and included in the North American Regional Broadcasting Agreement (NARBA). The method used followed that outlined in the Department of Transport Broadcast Specification No. 10. Basically it involves fitting the experimental points to the theoretical curve at a short distance from the transmitter where ground conductivity does not play an important part in determining field strength, and then observing the conductivity associated with the theoretical curve which most closely approximates the experimental points at greater distances. The resulting data have been published by Ireland [1961]. He gave the following conductivity values in millimhos/meter: Lake Superior, 7; Lake Huron, 10; Lake Erie, 10; and Lake Ontario, 15.

The author of this paper proceeded with laboratory measurements of the conductivity of the water samples. The conductivity of the water is dependent on temperature, the coefficient being 2.2 percent per degree Celsius. When the laboratory measurements were corrected to the measured temperature of the lakes, the following results were obtained: Lake Superior, 7.0; Lake Huron, 18.9; Lake Erie, 28.0; and Lake Ontario, 26.5.

The large discrepancies which existed between these values and those quoted by Ireland [1961] led to a reexamination of the theoretical curves on which the deduced conductivities were based, and to an additional experimental run on Lake Ontario.

2. Field Strength Calculations

A comparison was made between field strength calculations derived from three separate sources. For the single case of a dielectric constant $\epsilon=15$, and conductivity $\sigma=15$ mmho/m, data were taken from the Federal Communications Commission (FCC) curve drawn for 1000 kc/s. These curves are plotted in terms of millivolts/meter versus distance in miles, with the field strength adjusted to 100 mv/m at a distance of 1 mi. Since the field strength varies as $1/d$ over a plane, perfectly conducting earth, the field strength quoted in the FCC curves was multiplied by d and normalized to unity at short ranges. The result is a curve of attenuation relative to a plane, perfectly conducting earth. This form of presentation points up the effect of conductivity more clearly. For the same ground parameters and frequency, the attenuation of the field below that of a plane, perfectly conducting earth was also calculated using the curves and method outlined in the Summary Technical Report of the Committee on Propagation, NDRC [1946]. Finally, the same problem was treated as a summation of modes, following Bremmer [1949]. Five modes were summed to provide field strength values accurate from large distances in to about 60 mi. The mode numbers were calculated

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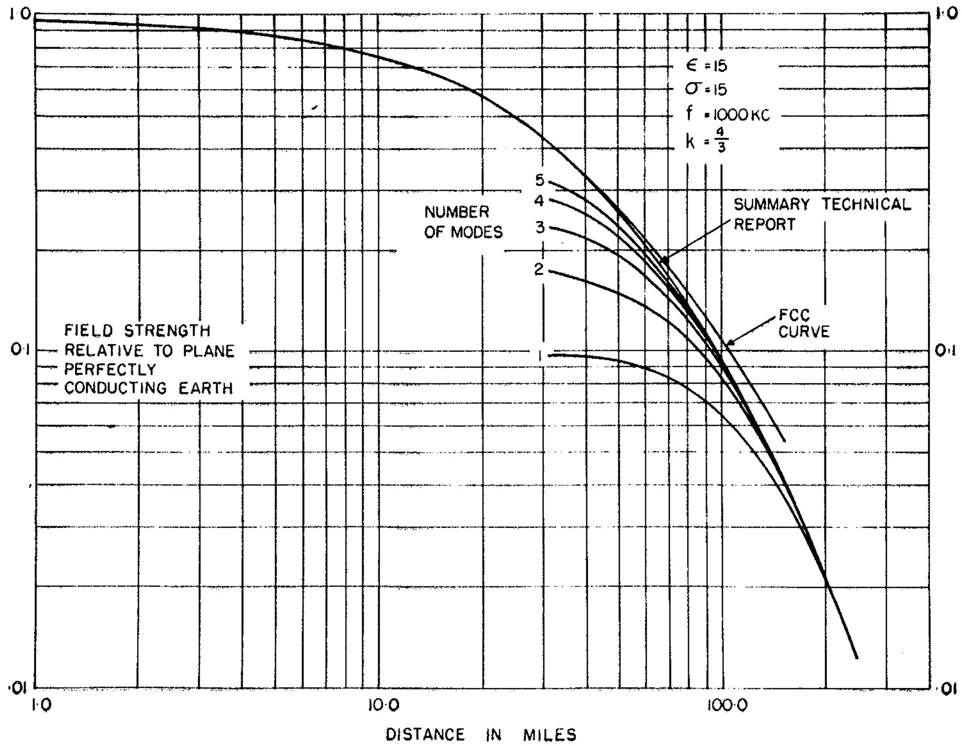

Figure 1. *Field strength versus distance curves.*
From (1) FCC Catalog, (2) Summary Technical Report, (3) Bremmer—summation of five modes.

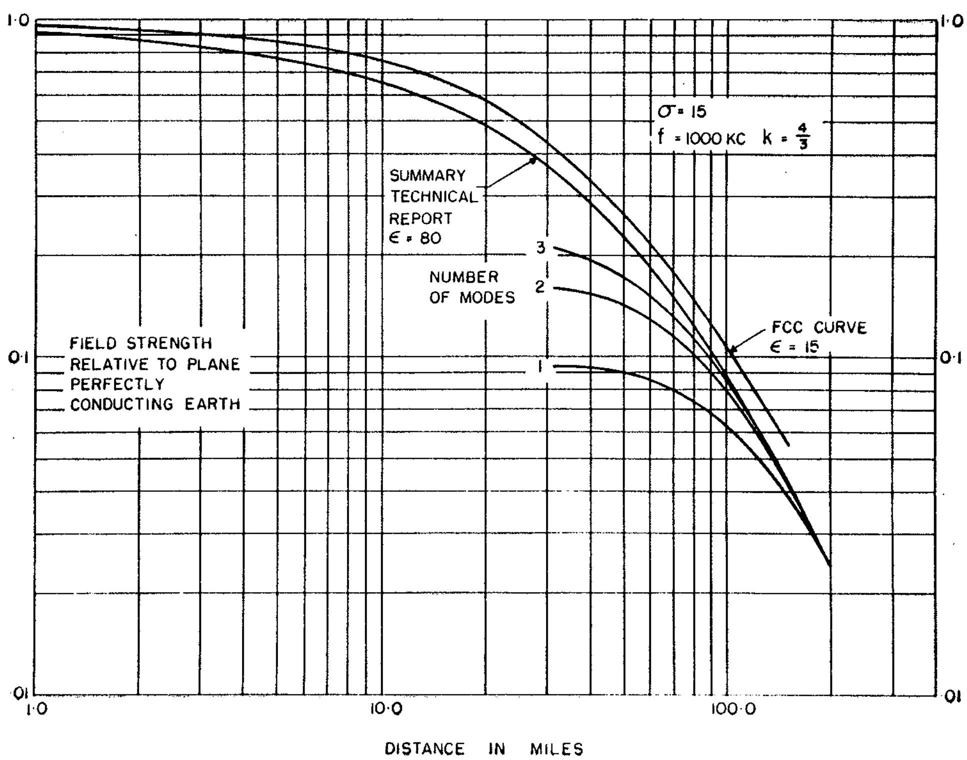

Figure 2. *Field strength versus distance curves.*
From (1) FCC Catalog, $\epsilon=15$, (2) Summary Technical Report, $\epsilon=80$, (3) Bremmer—summation of three modes, $\epsilon=80$.

from the expressions given in Bremmer's book, but use was made of the more accurate value of the limiting mode number for a flat earth, as given by Norton [1941].

Results of the three derivations of field strength curves are shown in figure 1. Up to about 20 mi the curves from the Summary Technical Report and the FCC are in close agreement. At distances beyond about 60 mi, the Summary Technical Report curve and that from Bremmer's work agree, and it is reasonable to assume, consequently, that the Summary Technical Report method is valid throughout the entire range of distances covered here. The FCC curve, on the other hand, departs steadily from the other curves as distance increases, and is about 2 db high at 150 mi.

Curves derived from the Summary Technical Report have also been compared with Bremmer's results for the two cases $\epsilon=80$, $\sigma=15$, $f=1000$ kc/s, and $\epsilon=80$, $\sigma=15$, $f=1200$ kc/s. In each case the agreement between the two curves is similar to that shown in figure 1. Consequently, for the comparison in the following section between theoretical and experimental data, the theoretical results from the Summary Technical Report have been used with confidence.

In Ireland's [1961] comparison of experimental data with theoretical curves, he actually used FCC curves for a dielectric constant of 15, instead of the more appropriate value of 80 for overwater transmission. In figure 2 is shown, for a frequency of 1000 kc/s—the frequency at which Ireland performed his experiment over three of the four Great Lakes—a comparison of the FCC curve for $\epsilon=15$, $\sigma=15$ and the Summary Technical Report curve for $\epsilon=80$, $\sigma=15$. The difference between the two curves extends to shorter ranges and consequently, if a deduction of conductivity is based on a short experimental run, the conductivity error is likely to be greater. It is obvious that some error must be assigned to Ireland's conductivity values as a result of the use of the inappropriate and inaccurate theoretical curves.

3. Field Strength Observations on Lake Ontario

Although some part of the discrepancy between conductivities deduced from field strength measurements and those measured in the laboratory may be assigned to the use of faulty theoretical curves, it is apparent that some additional factor is involved. As a result, field strength measurements were undertaken in June 1961. These were confined to Lake Ontario, and a map of the path taken is shown in figure 3.

A 150-ft insulated tower was erected at Point Petre at about 500 ft from the shore. Twenty-four radial ground wires, each 800 ft long, were laid on the ground with grounding rods at the ends. A number of western radials were run into the lake. The transmitter delivered a nominal power of 1 kw into this antenna and was operated at 1200 kc/s. Operation of the transmitter was limited to the daylight hours. For the first 3 hr of the run the transmitter was operated continuously, but for the remainder of the run it was turned off for 2 min every half hour for identification and to establish a noise level at the receiver. The transmitter was not modulated.

Receivers were carried aboard the NRC Motor Vessel "Radel II." A 15-ft whip was mounted on the wooden bridge of the vessel and connected through a matching network to two receivers. One receiver was a Stoddart field strength meter whose output drove an Esterline-Angus recorder. The

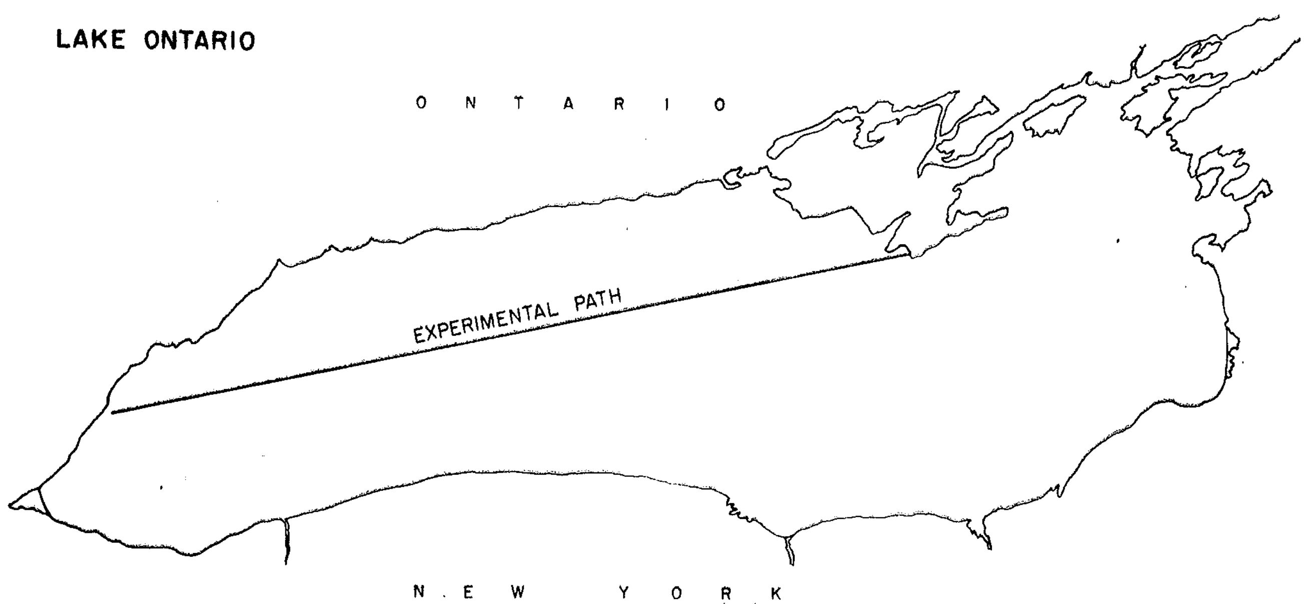

FIGURE 3. *Path of field strength measurements on Lake Ontario.*

other receiver had an attenuator at its input and was operated at a constant output level by varying the attenuation at the input. Both receivers were operated from an a-c voltage regulator. A signal generator was available and calibrations were made before and after each run, and spot checks were made on a number of occasions during the runs.

On the first run *Radel II* was taken to within 1000 yd of the beach. It then proceeded slowly (5 knots) out along the path shown in figure 3. After 4 mi the speed was increased to 9 knots, and this was maintained for the remainder of the run. In the early portion of the run, distances from the transmitter were obtained by radar to an accuracy of 25 yd. At about 5 mi from the transmitter, the radar echoes became blurred and uncertain as to exact reflection point. From this point on, distance was determined by dead reckoning until another radar fix was obtained off Toronto. The signal was steady through the first 100 mi of the run. For the last 25 mi, some variation in the recorded signal was observed. This amounted to about ±0.5 db. Some, and perhaps all, of this variation was due to interference from the Toronto region and the densely populated area on the northwest shore of the lake. The equipment worked well through this run and the data are believed to be reliable.

After a day's layover in Toronto harbour because of poor weather, a return trip was made over the same course. During the early part of this run much more interference was experienced than on the first run. The variability in recorded signal amounted to about 1.5 db. Although the weather was clear, a great deal of this variability was due to thunderstorm atmospherics. At about 90 mi from the transmitter this interference became negligible and was not a factor in the remainder of the run. This run, however, was not as satisfactory as the first. The calibration of the receiving equipment showed evidence of drift—the worst being a 1.5-db change in calibration observed at the end of the run. There was also a temporary failure of the ship's electrical supply during the run. As with the first run, the ship's speed was about 9 knots throughout most of the run, but was cut to about half this value through the 3.5 mi closest to the transmitter. The ship was edged in to a distance of 500 yd offshore, where a measurement was taken and the run terminated.

On both runs it was observed near the transmitter, where the communications receiver was being operated with a large amount of attenuation at the input, that a signal was being received through leakage into the case. As a result the data obtained on the communications receiver have not been used, and the experimental results given here are from the Stoddart record.

A total of nine stops was made during the two runs to collect water samples and to measure water temperatures. The temperature was measured to a depth of 50 ft, which was sufficient to determine the position of the thermocline.

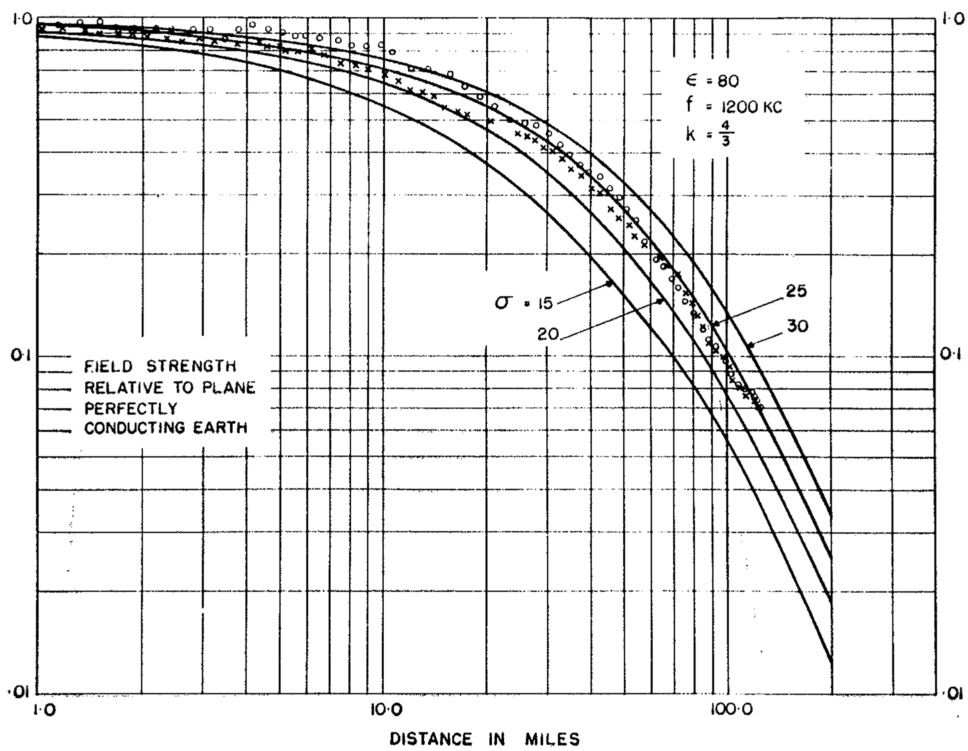

Figure 4. *Comparison of experimental and theoretical field strength versus distance curves.*

4. Data Reduction and Observations

The recorded output of the Stoddart field strength meter was read and was converted to an arbitrary decibel scale using the calibration data taken before and after each run. Small corrections were then applied as indicated by the spot checks of the calibration and tuning performed during the run. In order to convert these data to field strength relative to propagation over a plane perfectly conducting earth, a correction of 20 $\log(1/d)$ was applied to each reading. The readings were then normalized to fit the theoretical curves at ranges in the neighbourhood of one mile. The results are shown plotted in figure 4. The results for the outbound run fit the theoretical curves reasonably well. The points, with a few isolated exceptions, fall between the theoretical curves for conductivities of 20 and 25 mmho/m.

The curve for the inbound run is much less satisfactory. The manner in which it has been fitted to the theoretical curve is rather arbitrary since it does not conform in shape to the curve for any conductivity value. As mentioned in the previous section, however, experimental difficulties during this run were such that the results must be discounted.

The possibility was considered that a variation of water temperature with depth might produce a corresponding variation of conductivity sufficient to make a surface value inappropriate. Consequently temperature soundings were taken and the results are shown in figure 5. The first run shows a fairly well-developed isothermal layer from the surface to 25 to 40 ft. The temperature in this layer shows a small decrease along the path. There was quite an appreciable change two days later. The iso-

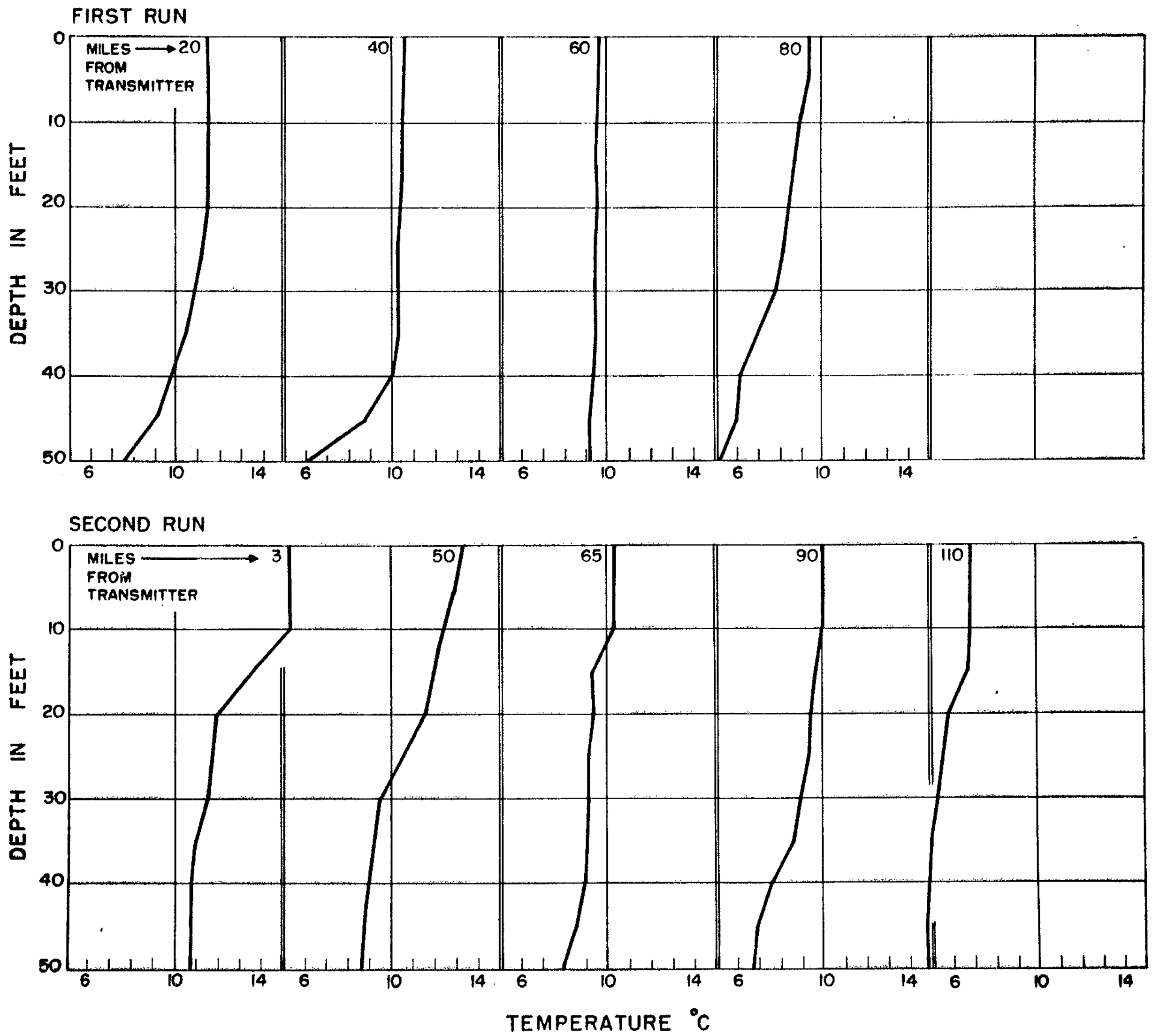

FIGURE 5. *Temperature soundings of Lake Ontario.*

thermal layer was not as well developed and there had been a marked increase in the east-west gradient of surface temperature. Based on laboratory measurements of the temperature dependence of the conductivity, a 20 percent difference in conductivity must have existed between the eastern and western ends of the path on the second run. However, it appears unnecessary to make any allowance for variations along the path or for the variation with depth, and consequently an average value of the temperature is used.

5. Electrical Conductivity of the Great Lakes From Water Samples

Water samples were taken from four of the Great Lakes (Superior, Huron, Erie, and Ontario) during the summer of 1959, and samples were taken from Lake Ontario during the summer of 1961. Measurements were made of the electrical conductivity of these samples and the results are shown in table 1. The measurements were made at room temperature (19 to 22 °C) and the results corrected to a common temperature of 21.5 °C. The average surface water temperature of Lake Ontario measured during the field trials described above is listed. The temperatures for the other lakes were measured by Ireland during the field trials leading to his paper of 1961. The last column of table 1 gives the conductivity

TABLE 1. *Lake conductivities, mmhos/m*

	Ontario	Erie	Huron	Superior
Conductivity at 21.5 °C[a]	29.2	26.7	18.2	8.4
Water temperature, °C[b]	10.8	23.7	23.0	14.2
Conductivity at measured temperature.	22.3	28.0	18.8	7.0

[a] Conductivity of water samples, corrected for temperature of 21.5 °C.
[b] Temperature of Lake Ontario measured during field trials described in this paper. Temperature of other lakes measured by Ireland, August 1959.

at these measured temperatures which represent the value which would be expected from the field trials. The value of 22 mmho/m for Lake Ontario is in good agreement with that determined from run 1 described above. The value 7 mmho/m for Lake Superior is also in agreement with Ireland's value. There is, however, considerable discrepancy for Lake Erie and Lake Huron between these values and the value of 10 mmho/m for both lakes quoted by Ireland.

The calculated contributions of the various ions to the total conductivity are listed in table 2. These

TABLE 2. *Contribution of the various ions to conductivity* (21.5 °C)

Ion	Erie	Huron	Superior
HCO_3	5.8	4.1	2.6
Ca	10.1	7.3	3.8
Mg	3.6	3.2	2.4
Na	1.8	0.6	0.2
Cl	3.9	1.2	0.2
SO_4	3.4	1.8	0.3
Calculated conductivity	28.6	18.2	9.5

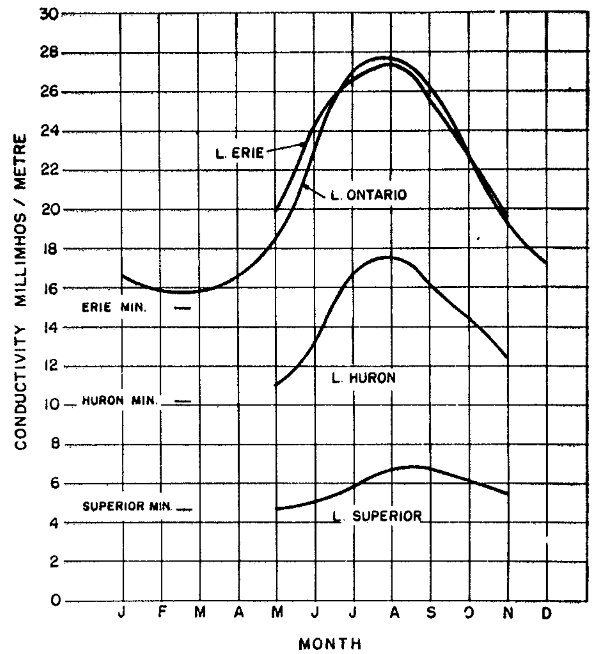

FIGURE 6. *Seasonal variation of conductivity of Lakes Ontario, Erie, Huron, and Superior.*

calculated conductivities are in good agreement with the measured values, and show clearly the effect of the change from Pre-Cambrian rock in the drainage area of Lake Superior to the sedimentary rocks around the Lower Lakes. There is no evidence in these figures of industrial pollution affecting the conductivity of the Lower Lakes, as has been suggested [Ireland, 1961].

Laboratory measurements of the dependence of conductivity on water temperature have yielded a relationship

$$\sigma_{T\,°C} = \sigma_{20\,°C}[1+0.022(T-20)].$$

Using this relationship, the conductivity of the Great Lakes as a function of the time of year has been calculated. The temperature of the surface water of the Lakes has been taken from a paper by Miller [1952]. There is, of course, a variation in temperature from year to year, but these curves will be indicative of the conductivity change that occurs. Except for Lake Ontario, water temperatures were not quoted for the winter months. However, since the Lakes usually do not freeze, but do approach freezing temperature, a fairly accurate estimate of their minimum temperature may be made. The corresponding minimum value of conductivity is shown in figure 6, together with the plot of conductivity for the months when temperature values are given.

6. Discussion

In the process of determining the radiowave conductivity of the waters of the Great Lakes it has been discovered that, in a limited number of cases checked, discrepancies exist between propagation curves deduced from the work of Bremmer and those published by the Federal Communications Commission. A better agreement exists between the former and those calculated from the Summary Technical Report. The error is more likely to reside in the FCC curves, but a much more extensive set of calculations would be necessary to determine the extent of the errors.

The electrical conductivity of the waters of Lakes Superior, Huron, Erie, and Ontario has been determined by laboratory measurements of water samples. By measuring radio field strength as a function of distance on Lake Ontario it has been shown that the laboratory-measured conductivity may be applied to the description of radiowave propagation on this lake. By inference the water sample conductivities of the other lakes are believed to be equally valid for radio propagation purposes. Over shallow lakes the penetration of the propagating radiowaves could be sufficient to make the lake bottom a significant factor in an effective conductivity. In the Great Lakes this effect will not be significant at broadcast band frequencies and higher, except near the shore. At 500 kc/s, and taking the lowest conductivity value (March) for each lake, the penetration depths (field $1/e$ of the surface value) are 30 ft for Lake Superior, 25 ft for Lake Huron, and 20 ft for Lake Erie and Lake Ontario. The penetration depths are, of course, less for higher frequencies and other months of the year. The depths of all the lakes except Lake Erie are very much greater than these penetration depths. The average depth of Lake Erie is about 60 ft, or three times the penetration depth. Thus even for Lake Erie the lake bottom should have no effect on propagation in the broadcast band, although the effect may become noticeable in certain areas of the lake at not much lower frequencies.

The conductivities of Lakes Huron, Erie, and Ontario, as stated in this paper, are significantly different from those recorded in the radio literature [Fine, 1954, and Ireland, 1961]. In addition, the variation of conductivity with temperature and consequently with time of year is considerable. The variation is such that for a 100-mi path on Lake Ontario the field strength of a 1 Mc/s signal will be 7 db higher at the maximum in August than that at the minimum in March. Although the conductivity values given in this paper differ from those published in radio conductivity maps, they are in accord with values well known to geologists. D. V. Anderson, Department of Geological Sciences, University of Toronto, lists almost identical conductivities in a private communication [1960], and similar values are given in a report by Thomas [1954].

In the calculations of field strength the effect of the atmosphere was included through the use of the effective earth's radius, $k=4/3$. The effect of the atmosphere is very small, however, as may be seen by considering the shadow factor—the factor by which the field strength is reduced due to the earth's curvature. Thus over water with a conductivity of 20 mmhos/m the shadow factor is 0.80 at 100 miles when $k=4/3$, and is equal to 0.74 when $k=1$. Therefore the difference between the assumed atmosphere and a homogeneous atmosphere under these conditions is only 0.7 db. The accuracy of the measurements was obviously not sufficient to distinguish between the standard atmosphere of $k=4/3$ and any other reasonable atmospheric gradient.

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