

Radio communication in the sea

Oceanography and the nuclear submarine have spurred interest in very-low-frequency propagation in the sea. A comprehensive survey is offered of what is feasible in the field of underwater communications

Richard K. Moore

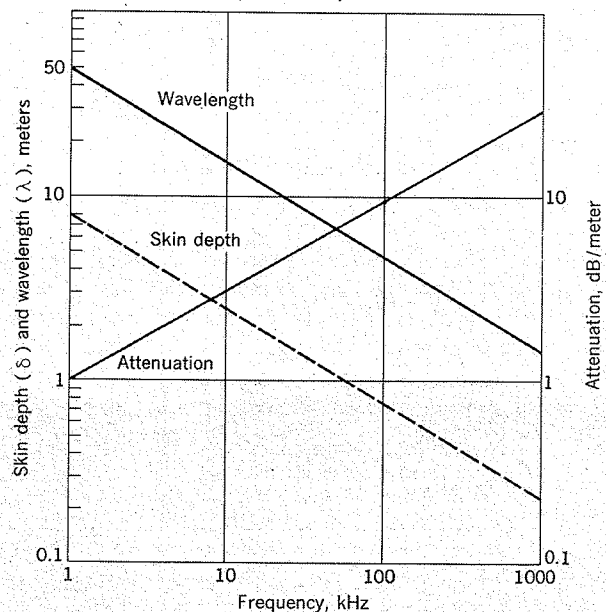
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Because of the electromagnetic properties of seawater, very-low-frequency communication systems are used. Surface-to-submarine, submarine-to-surface, and submarine-to-submarine propagation, as well as antennas and noise, are considered. It is shown that seawater is a good conductor and that atmospheric noise is generally more of a problem in sea communications than thermal noise. Communication ranges are limited by the effects of depth attenuation and atmospheric noise. Ranges in the tens of kilometers are possible for antennas within 5 meters of the surface; bandwidth must be less than 1 Hz and tens of kilowatts of power are required.

Since submarines and radio communication became practical at nearly the same time, it is not surprising that attempts to use radio for communication with and between submerged submarines have been made almost from the beginning of radio. At various times, experiments have been performed to show that submerged antennas are superior even for communication in air—but the results, when properly evaluated, have always been that submerged antennas are much worse. The advent in the 1950s of nuclear submarines that did not have to charge batteries gave a new impetus to the study of submarine communications since opportunities for communication with a submarine, on the surface for battery charging (or near the surface and snorkeling), now became rare. During that period also, communication between subterranean installations became a problem, and subterranean communications studies were initiated. Today, as exploitation of resources on the ocean bottom appears likely to boom, the communication problem once again becomes important.

Seawater is a poor medium for communications because it is a good conductor. Point-to-point communication within the sea is limited to such short ranges by attenuation that even for distances of a few kilometers the waves travel to the surface, are refracted, travel along the surface, and are re-refracted into the sea. Very-low frequencies (VLF) must be used to achieve any significant

FIGURE 1. Skin depth, wavelength, and attenuation in seawater ($\sigma = 4$ mhos per meter).



World War II: Electronics and the U.S. Navy (page 56)

Gordon D. Friedlander has been a staff writer for IEEE SPECTRUM for the past four years. This is his second historical article on military electronics, with particular emphasis on naval applications during the Second World War. A biographical sketch of Mr. Friedlander appears on page 111 of the February 1965 issue.

In the preparation of the two-part article, starting in this issue, and his feature piece "World War II radar: The yellow-green eye" (IEEE SPECTRUM, May 1966), Mr. Friedlander received very helpful technical background information, historical records, and excellent action photographs from **Rear Admiral Ernest M. Eller, USN (Ret.)**, Director of Naval History for the Department of the Navy.

Admiral Eller was graduated from the U.S. Naval Academy in 1925, and he holds the M.S. degree in psychology from George Washington University. During World War II, he served with distinction in various staff and command assignments with the U.S. Pacific Fleet. The Legion of Merit with Combat "V" and the American Defense Service Medal are among his service awards. He is the author of numerous wartime technical reports and naval papers, and a frequent contributor to the *U.S. Naval Institute Proceedings*.



Information theory and cybernetics (page 75)



H. Marko (SM) has been professor of Communications Techniques at the Technische Hochschule, Munich, F.R. Germany, since 1962. He is also director of the school's Institute for Communications Techniques and, since 1965, he has been a member of the Administrative Committees of the Society for Communications Techniques (VDI) and of the German Society for Cybernetics.

He received the diploma in communications techniques and the Dr. Ing. degree from the Technische Hochschule Stuttgart, Germany, in 1951 and 1953 respectively. In 1953 he joined the Standard Elektrik Lorenz AG as a development engineer. There he founded and directed the Department of Fundamentals for Transmission Techniques, and he has worked on projects concerning telegraphic and radio techniques.

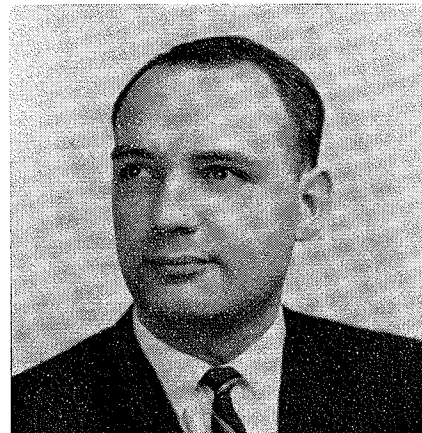
In 1959 he became chairman of the Committee on Information and Systems Theory of the Society for Communications Techniques within the Verband Deutscher Ingenieure. He has lectured at the Technische Hochschule Stuttgart and at Technische Hochschule Karlsruhe.

Computer-aided design (page 84)

Ronald A. Siders is a consultant in the manufacturing area on the staff of the management consulting firm of Booz, Allen & Hamilton, Inc., Cleveland, Ohio.

He was graduated from the Pennsylvania State University in 1959 with a bachelor of science degree in earth sciences, and was awarded the master of business administration degree with distinction from the Harvard Business School in 1966. Between 1962 and 1964, as a member of the Programming Systems Division of Honeywell Inc., he was engaged in work on the development of Fortran compilers and numerical control programming systems. He also served as a member of the Honeywell staff during the summer of 1965.

Since joining Booz, Allen & Hamilton, Inc., Mr. Siders has conducted a variety of assignments in computer systems design and installation, organization studies, and executive compensation. He is the lead author of *Computer Graphics—A Revolution in Design*.



Future goals of engineering in biology and medicine (page 93)

Nilo Lindgren A biographical sketch of Mr. Lindgren appears on page 196 of the March 1965 issue.

penetration into the sea, and information rates for submarine communication are therefore very low. Furthermore, atmospheric noise is extremely high at these low frequencies, necessitating high power even for low data rates. Actually, communication with men on the moon will be far easier than communication with a deeply submerged submarine less than 100 km away.

Electromagnetic properties of seawater

The electromagnetic properties of seawater limit communication ranges. To see this, consider a plane wave traveling in the sea, with electric field E given by

$$E = E_0 e^{j\omega t - \gamma x} \quad (1)$$

The medium properties are contained in the propagation constant γ :

$$\gamma = \sqrt{j\omega\mu(\sigma + j\omega\epsilon)} = \alpha + j\beta = \frac{1}{\delta} + j\beta \quad (2)$$

If a high enough frequency were used that the displacement current greatly exceeded conduction current, the frequency-independent value of α would become

$$\alpha = \frac{1}{\delta} = \frac{\sigma}{2} \sqrt{\frac{\mu}{\epsilon}} \quad (3)$$

A typical conductivity σ is 4 mhos per meter and permittivity ϵ is $81 \epsilon_0$, resulting in an α of 84 nepers per meter. The skin depth δ is therefore only 1.19 cm, and the penetration at such frequencies is essentially negligible. For this reason, low frequencies must be used in any attempt at communication through the sea.

At low frequencies, where displacement current may be neglected, the propagation constant is approximately

$$\begin{aligned} \gamma &= \sqrt{j\omega\mu\sigma} = \sqrt{\frac{\omega\mu\sigma}{2}} (1 + j) \\ &= \alpha + j\alpha = \frac{1}{\delta} + j\frac{1}{\delta} = \beta + j\beta \end{aligned} \quad (4)$$

The amplitude of the wave therefore decreases with the

distance in proportion to $e^{-x/\delta}$.

Figure 1 shows the variation of skin depth with frequency for the typical conductivity value of 4 mhos per meter. Because attenuation for one skin depth is 8.7 dB, no communication system can stand a path many skin depths long; at 10 kHz the attenuation is 87 dB for a distance of only 25 meters. Even at a frequency as low as 100 Hz, the distance for 87-dB attenuation is 250 meters, in addition to spreading attenuation and refraction losses.

Because phase and attenuation constants are the same in a conducting medium, a skin depth is also the distance for a radian of phase shift, and the wavelength is just 2π skin depths; hence wavelength is also given in Fig. 1. The vast difference between wavelength in sea and in air at low frequencies is often not fully appreciated. Note that at 10 kHz, the wavelength in air is 30 km, whereas it is only 15.7 meters in the sea. In a conducting medium, the wavelength is the distance for 2π nepers, or 55 dB of attenuation.

Sea conductivity, normally considered to be 4 mhos per meter, is in fact both temperature and salinity dependent. Thus σ varies from less than 2 in the cool and not-very-saline Arctic to 8 or more in the warm and highly saline Red Sea. Attenuation in the Arctic is therefore less than in temperate zones, so communication is feasible at greater depths or ranges. In land-locked hot seas, however, the situation is reversed.

Electromagnetic communication experiments in the sea were conducted as early as the turn of the century.¹ During World War I, both the German and Allied navies conducted experiments with low-frequency submerged reception, using low frequencies to transmit to submerged submarines.²⁻⁵

The theory of communication with submerged terminals was not developed until much later,⁶ but the necessary tools were provided by Sommerfeld in 1909⁷ in his classic paper dealing with propagation over the earth and sea. The history of above-sea propagation theory need not be related here; it covers, of course, hundreds of papers over the years since 1909. The theoretical work on the submerged propagation problem has been summa-

ized, with historical notes, in a monograph by Banōs⁸ in which numerous contributions by Wait are mentioned, along with Banōs' early work and that of McCracken.

Coupling of circuit and wave in a conducting medium has occupied numerous workers since the beginnings of experimental work. The early papers referred to were as concerned (or more so) with antennas as with propagation. Braun's early experiments with separated electrodes have been repeated, in one form or another, many times since then. Bouthillon's loop antenna, as with other loops used on submarines since World War I, was a magnetic dipole. Willoughby and Lowell's loop antenna, however, was in fact an electric antenna; they neglected to take into account the decrease in wavelength from air to sea, making their antenna many wavelengths in circumference with current distributions different from those assumed.

Much excellent work on the submerged antenna problem was done by the late O. Norgorden of the U.S. Naval Research Laboratory. Unfortunately, his reports are not widely available. Moore, in a 1951 dissertation, outlined problems of antenna analysis in conducting media, treating both small loops and insulated dipoles. The differences between antenna analysis in air and water were abstracted in a paper in 1963.⁹ In the late 1940s, Tai published a number of reports in the Harvard University Cruft Laboratory series dealing with the submerged antenna problem. Wait has made numerous contributions to the theory of submerged antennas; King and Iizuka have published a series of papers detailing careful measurements of impedance for several types of submerged antennas. Other contributors too numerous to record here have added to both the theoretical and experimental literature on this subject since 1960. A number of pertinent papers may be found in a special issue of IEEE TRANSACTIONS ON ANTENNAS AND PROPAGATION, published in May 1963 (vol. AP-11).

The great concern in VLF propagation in recent years has been the combined result of submarine and subterranean communication interest. There have been many papers on propagation mechanisms and atmospheric noise. Anyone seriously interested in *long-distance* submarine-subterranean communication should search this recent literature.

Propagation

Communication with submerged stations involves propagation to and above the surface, even when both terminals are submerged, owing to the extremely high attenuation for direct waves between two submerged stations. We may therefore consider the following cases of communication with submerged terminals:

1. Surface to submarine.
2. Submarine to surface.
3. Submarine to submarine (via surface).

Case 3 is a combination of cases 1 and 2.

Practical transmitting antennas in the air at VLF produce vertically polarized waves. Although these waves have curved fronts, for the purpose of treating their refraction into the sea the waves may be considered plane, an adequate approximation in any local region remote from the source. If the sea were perfectly conducting, only a vertical component of electric field would be possible in the air, as in Fig. 2(A). Because the intrinsic impedance for waves in sea is much less than that for waves in air, the actual situation is not much different from that of

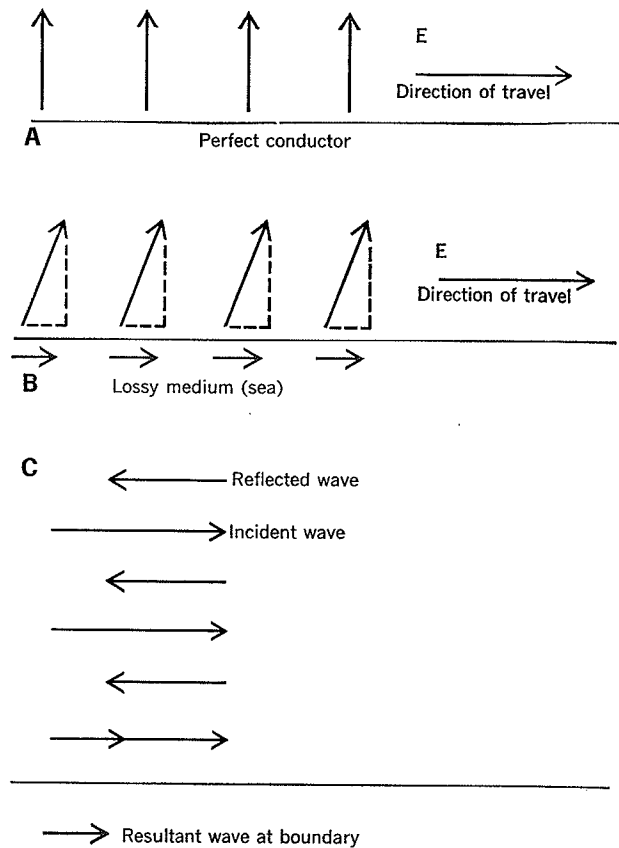


FIGURE 2. Waves in air over sea. A—Wave over perfectly conducting boundary. B—Wave over lossy medium. C—Normally incident wave near air-sea boundary.

FIGURE 3. Mode of travel of waves from transmitter that is submerged. A—From submarine to surface. B—From submarine to submarine.

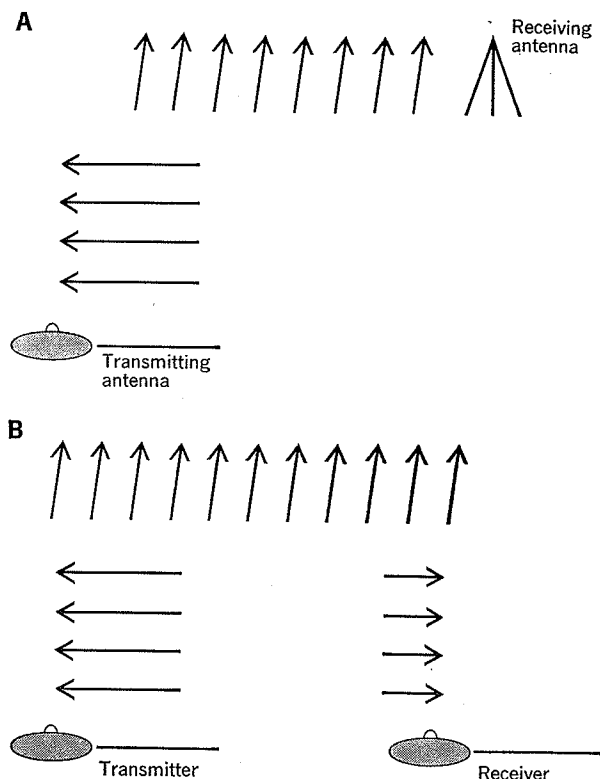


Fig. 2(A). In Fig. 2(B) a slight forward tilt of the electric field vector is indicated. The horizontal component is due to energy propagating into the sea; because it is slight, only a small part of the total power transmitted past an incremental area on the surface is diverted into the sea. If we were concerned with communication in air, we would consider this to be a loss; because we are concerned with propagation into the sea, this component is the one in which we are interested. If we could somehow launch from the ionosphere a vertically traveling horizontally polarized wave having the same incident electric field strength, as in Fig. 2(C), the reflection coefficient for the wave in the sea would be the same size as for the horizontally traveling vertically polarized wave.

Assume we launch a plane wave from beneath the surface of the sea and transmit it vertically; a small part of it would be transmitted through the surface and travel up in the air as a horizontally polarized wave. The same fraction crosses the boundary as is transmitted into the sea for a downcoming horizontally polarized wave. If this ideal plane-wave source were to transmit up at a slightly nonvertical angle, however, the wave would be refracted into a vertically polarized, almost horizontally traveling, wave; the discontinuity is so great that the angle of refraction is almost 90 degrees, even for very small angles of incidence. With practical horizontal antennas, which must be close to the surface because of attenuation in the sea, most of the energy is radiated at angles such that this refraction occurs (see Fig. 3). The wave set up above the surface is the same as that of a quadrupole radiator located above the surface and made up of two side-by-side antiphased vertical dipoles. Hence, at long distances it may be treated like a wave radiated in air, and the energy that leaks back into the water may be described by the mechanism of Fig. 2(B). Figure 3(B) shows this "up-over-and-down" mechanism for communication between two submerged antennas.

Submerged vertical dipoles, because of the null in their pattern in the direction of the dipole, are not as effective as horizontal dipoles. Because there are no strong components of the radiation directly upward, most of the energy must travel a longer path through water and also suffer a greater refraction loss. We therefore treat here only horizontal dipoles (electric and magnetic).

Surface to submarine—mathematical form

For a wave traveling in the y direction in air, the electric (\mathbf{E}_a) and magnetic (\mathbf{H}_a) fields are given by

$$\begin{aligned}\mathbf{E}_a &= \mathbf{1}_z E_0 e^{j(\omega t - \beta y)} f(\text{distance}) \\ \mathbf{H}_a &= \mathbf{1}_x \frac{E_0}{\eta_a} e^{j(\omega t - \beta y)} f(\text{distance})\end{aligned}\quad (5)$$

where $f(\text{distance})$ is a suitable function of distance. Although a pure plane wave would vary with distance only exponentially, the wave over the earth represented locally here by plane components may have other distance factors; these are the subject of numerous papers and books on propagation in air. The form of this variation need not concern us here. Unit vectors $\mathbf{1}_z$ and $\mathbf{1}_x$ are in the vertical and transverse directions respectively. Intrinsic impedance η takes its air values, as indicated by subscript a ; since phase-shift constant β is generally used only with its value in air, the subscript is usually omitted.

The value of \mathbf{E}_a given in (5) is an approximation for a

perfectly conducting sea. To obtain the horizontal component of \mathbf{E} , not present with perfect conductivity, note that the magnetic field is tangent to the air-sea boundary; because there is no permeability difference, it must be the same on both sides:

$$\mathbf{H}_s = \mathbf{H}_a \text{ at air-sea boundary} \quad (6)$$

The horizontal component of \mathbf{E} in the sea (and in air as well) is related to \mathbf{H}_s by

$$\mathbf{E}_s = \mathbf{1}_y \eta_s \mathbf{H}_a \quad (7)$$

Writing this in terms of E_0 and giving the complete expression for a point at depth d_r (whose z coordinate is $-z_r$), we find the expression for the field in the sea:

$$\mathbf{E}_s = \mathbf{1}_y \frac{\eta_s}{\eta_a} E_0 e^{j(\omega t - \beta x - \alpha d_r)} e^{-\alpha d_r} \quad (8)$$

The ratio of the intrinsic impedances in sea and air, the refraction-loss ratio, is very important; it is given by

$$\frac{\eta_s}{\eta_a} = \sqrt{\frac{j\omega\epsilon_0}{\sigma}} = \sqrt{\frac{j}{g}} \quad (9)$$

where the significant ratio of conduction current to air-dielectric displacement current is labeled g . Practical values of g are quite large; for example, it is 7.2×10^6 at 10 kHz. Thus the refracted \mathbf{E} (or the horizontal component of \mathbf{E} in air) is less than 1/1000 of the value of the magnitude of \mathbf{E} for the wave in air. Although the magnetic field is the same in sea and air, the Poynting vector in sea is reduced by the refraction ratio.

Because the magnetic field is unaffected by refraction, one might be led to believe that a magnetic receiving antenna is inherently superior to an electric receiving antenna in this case. This is not true, however, for antennas of comparable size extract roughly the same power from the wave whether they are primarily sensitive to the electric or magnetic fields; the Poynting-vector value determines how much signal can be received for a given size antenna, not the relative value of the \mathbf{E} - or \mathbf{H} -field alone.

Submarine to surface—mathematical form

The fields of a submerged dipole may be obtained by modifying the Sommerfeld cylindrical-coordinate boundary value problem approach. The resulting expressions are, in general, quite complex, and have been well reported in the literature.^{6,8} A few expressions will be given here to indicate the type of variation in the far field.

Banòs identifies four regions: (1) asymptotic range, (2) intermediate range, (3) near-field range, and (4) vicinity of vertical axis. His intermediate and asymptotic ranges (jointly called "far" by Moore and Blair) are at distances significantly greater than the free-space wavelength/ 2π . In these regions the field is fairly simple, and is easily represented in local vicinities by a plane wave, as in the case of a surface antenna. In regions 3 and 4 the situation is more complicated (as it is for an antenna in air), and distance variations of different \mathbf{E} and \mathbf{H} components are not alike. Consequently, we restrict ourselves here to the far-field region, where the field is

$$\begin{aligned}\mathbf{E}_a &= \mathbf{1}_z \frac{60\beta_a I l e^{j(\omega t - \beta_a r)}}{r} e^{-j d_i/\delta} F\left(1 - \frac{j}{\rho}\right) \\ &\times \left[\frac{-\sqrt{-j}}{\sqrt{g}} e^{-d_i/\delta} \cos \phi \right] \quad (10)\end{aligned}$$

where I is antenna current, l is effective length of antenna, r is horizontal distance, d_t is depth of transmitting antenna, ϕ is the angle, measured in horizontal plane, from a zero along direction of submerged horizontal antenna, F is ground-wave distance attenuation factor, and ρ is the horizontal distance in units of free-space wavelength/ 2π .

The factor in square brackets is due to the source being submerged; the remainder of the equation is the same as for a vertical dipole in air. A readily available reference for the ground wave in air can be found in Jordan.¹⁰ Except for the submerged-antenna factor, Eq. (10) is the same as Jordan's Eq. (16-13) with suitable approximations made for high conductivity and for a dipole at the surface. F is a complex function of the "numerical distance" ρ ; it is plotted in Jordan and elsewhere.

The submerged-source factor contains four components

$$\begin{array}{ccccccc} (-\sqrt{-j}) & \left(\frac{1}{\sqrt{g}}\right) & (e^{-d_t/\delta}) & (\cos \phi) & (11) \\ \text{Phase} & \text{Refraction} & \text{Depth} & \text{Quadrupole} & \\ & & \text{attenuation} & & \end{array}$$

Note that the refraction factor for the wave *leaving* the sea is the same as that for a wave *entering* the sea. The cosine variation is that for a quadrupole; hence, the submerged horizontal antenna may be thought of as equivalent to a surface vertical quadrupole. The depth factor is the same as for a wave entering the sea. Presence of depth attenuation, the same as for a wave traveling vertically from antenna to surface, indicates that the mode of transmission for any other path between antenna and surface would be longer, with greater attenuation.

In the near-field region the depth attenuation is the same. Distance variation is different, and other components become significant—each component having its own refraction factor and angular variation.

Submarine to submarine—mathematical form

When both terminals are submerged, the near- and far-field regions may be described by one set of electric field expressions. Although these expressions are given with the ground-wave-attenuation factor included, practical ranges for two submerged terminals are so small that F may be considered always unity. These field expressions are given in the form most suitable for far-field use:

$$E_r = \frac{60\beta I l}{r} F \left\{ 1 - \frac{j}{F\rho} (2F - 1) - \frac{2}{\rho^2} \right\} \times \left[\frac{1}{g} e^{-(d_t+d_r)/\delta} \cos \phi \right] \quad (12)$$

$$E_\phi = \frac{120\beta I l}{r} F \left\{ \frac{1+F}{2\rho} - \frac{jF}{2\rho^2} \right\} \left[\frac{1}{g} e^{-(d_t+d_r)/\delta} \sin \phi \right] \quad (13)$$

In the far-field region, E_r is predominant because $1/\rho$ is small compared with unity. This means that the field of a horizontal dipole is best received by another submerged horizontal dipole aligned with the first, end to end. For a direct path, this would of course be a poor alignment, for the patterns of the dipoles would have nulls in each other's direction; with the up-over-and-down mode, however, this alignment is best.

In (12), quadrupole angular variation is evident, and

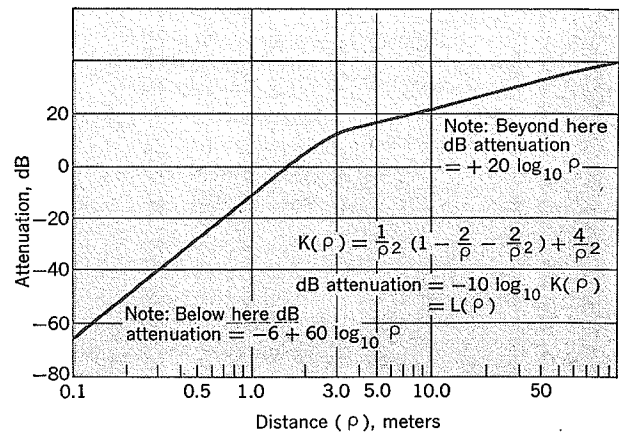


FIGURE 4. Field variations for an electric antenna.

the refraction factor is squared because it occurs both on emergence of the wave into the air and on re-refraction into the sea. Depth attenuation is the sum of the attenuations (product of the exponentials) for the upward transmitted wave and the downward received wave. Its form shows the presence of the up-over-and-down mode.

At shorter ranges ($\rho < 1$) the two components are comparable in size, with E_ϕ reaching a maximum broadside to the antenna. Because of its direction, it also corresponds with a receiving antenna parallel to the transmitting antenna. In the near-field region, the distance variation is as the inverse cube, as for a static dipole. Figure 4 presents an example of field variations, showing the transition from near- to far-field regions. It is self-explanatory.

Although it is usually easier to build effective electric-dipole antennas, there are times when magnetic dipoles may be more appropriate. Fields of magnetic dipoles exhibit most of the characteristics already described for electric-dipole fields. The strongest component in the sea of the electric field of a horizontal magnetic dipole, in the far-field region, is radial; the depth attenuation factors are the same, indicating the same up-over-and-down mode of propagation; and the refraction factors are the same. Maximum radiation, however, is (in the far field) at right angles to the dipole, that is, in the plane of the loop that normally is used for a magnetic dipole. Fields beneath the surface for submerged transmitting horizontal magnetic dipoles are given by

$$E_r = \frac{60\beta^2 I S N}{r} F \left\{ 1 - \frac{j}{\rho} - \frac{1}{\rho^2} \right\} \times \left[-\sqrt{j} \frac{1}{\sqrt{g}} e^{-(d_t+d_r)/\delta} \sin \phi \right] \quad (14)$$

$$E_\phi = \frac{120\beta^2 I S N}{r} F \left\{ 1 - \frac{j}{\rho} - \frac{j}{\rho^2} \right\} \times \left[-\sqrt{-j} \frac{1}{g} e^{-(d_t+d_r)/\delta} \cos \phi \right] \quad (15)$$

$$H_\phi = \frac{60\beta^2 I S N}{\eta_0 r} F \left\{ 1 - \frac{j}{\rho} \right\} [e^{-(d_t+d_r)/\delta} \sin \phi] \quad (16)$$

$$H_r = \frac{j120\beta I S N}{\eta_0 r} F \left\{ \frac{1}{\rho} - \frac{j}{\rho^2} \right\} [e^{-(d_t+d_r)/\delta} \cos \phi] \quad (17)$$

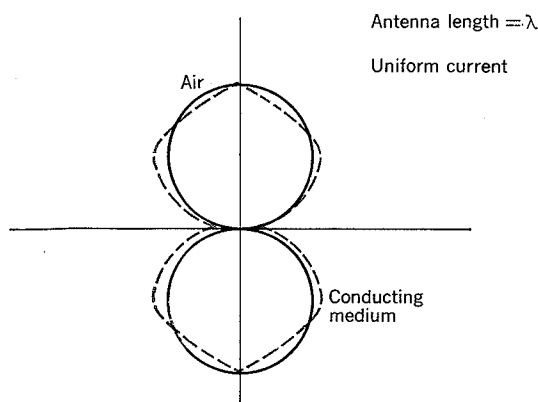


FIGURE 5. Effect of coordinate origin on radiation pattern.

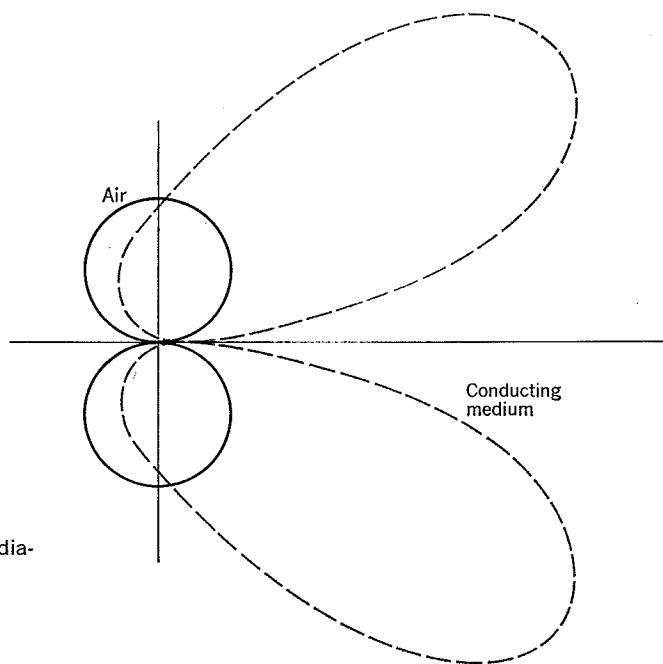


FIGURE 6. Submerged insulated antennas. A—Flat-plate termination. B—Spherical termination. C—Trailing-wire configuration.

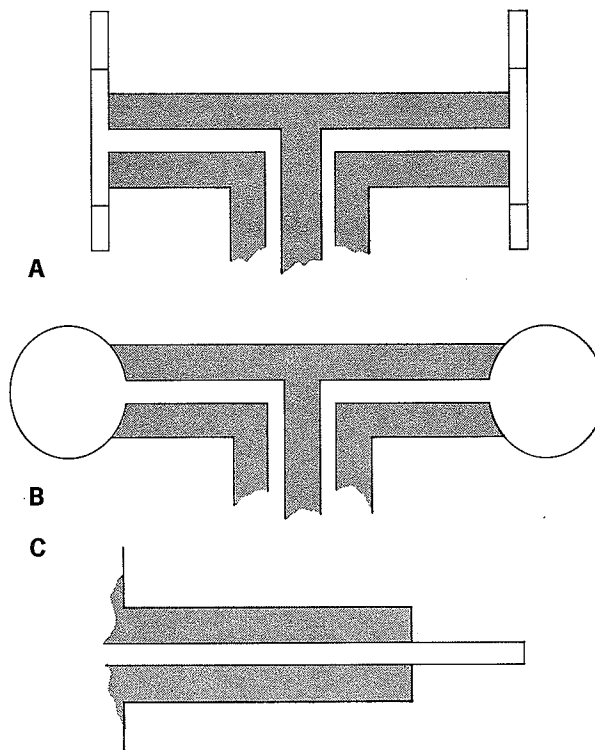
The dipole is represented by a planar loop of area S , with N turns carrying current I .

Antennas

Waves are coupled to current in circuits by antennas. All of the preceding propagation expressions have been predicated on dipole sources. In air, more elaborate sources are not feasible at the low frequencies necessary for submarine communication. Nevertheless, these electric and magnetic dipoles in the sea take on interesting forms different from those common in air, primarily because attenuation and phase shift are equal in the conducting medium; in air, attenuation between the parts of an antenna is negligible.

The differences between antennas in air and in conducting media have been discussed in detail earlier by Moore.⁹ An illustration of problems that arise is shown in Fig. 5. In air, because of no attenuation, antenna patterns or polar diagrams are normally used to illustrate directional properties; the location of the origin of coordinates used in calculating these diagrams is immaterial as long as it is somewhere near the antenna. When one attempts to make a similar calculation for an antenna in the sea, he discovers that the choice of an origin of coordinates is all-important, for the pattern depends on it, as shown in the illustration. The reason is that attenuation between one side of the antenna and the other is so great that the major contribution to the field at any point is primarily due to the nearest part of the antenna. The net result is that antenna patterns are meaningless in conducting media.

Electric antennas in conducting media may take the same form as short electric antennas in air, but the rapid decay of antenna current along a conductor suggests that there must be a better approach. The approach



commonly used is to send current through insulated wires to electrodes or contacts within the seawater. A current then flows between these contacts and radiates. Antennas in air really do much the same, except conduction current flows in the sea whereas displacement current flows in air; further, insulation is unnecessary in air.

Figure 6 shows several ways to make an electric antenna for the sea. A dipole with flat plates to contact the water is shown in Fig. 6(A). If the plates are large compared with the insulation diameter, which is small rela-

tive to wavelength in the sea, a current is set up in the sea equal and opposite to that in the wire, and the electric fields in sea and wire are parallel. This antenna may be analyzed as a coaxial transmission line, with the seawater forming the outer conductor. Another way to terminate this "transmission line" is shown in Fig. 6(B); spheres and cylinders have both been used in the configuration.

The antenna form of Fig. 6(C) is commonly used as a trailing wire. It was called "short-circuit coaxial" by Moore, with the bare end serving to short-circuit the "transmission line." Because a trailing wire can be very long, this is often the most practical form for a submarine antenna. The antenna was independently analyzed by Norgorden and by Moore. The wavelength in the coaxial antenna is intermediate between that in sea and that in air; consequently, an antenna that is long in terms of sea wavelengths may still be a small fraction of a wavelength long as far as the coaxial transmission line mode is concerned. Any of the antennas of Fig. 6 may carry uniform currents even if they are much longer than a sea wavelength, whereas long antennas with uniform currents are not feasible in air.

If one assumes that any of the techniques shown in Fig. 6 short-circuits the equivalent coaxial transmission line, the input resistance R_{in} is given by

$$R_{in} = \omega\mu l/8 \quad (18)$$

where l is the length of the antenna. At first it seems surprising that the diameter of the antenna does not enter into the resistance. This is explained by noting that the current spreads out over a large cross section determined by the water wavelength, and the cross section of the insulated section detracts a negligible amount from the total cross-sectional area; however, the resistance is very low. For example, a 100-meter-long antenna at 10 kHz has a resistance of only about an ohm.

The reactance of an antenna of this type is given by

$$X_{in} = \frac{\omega\mu l}{2\pi} [0.116 + \ln(\delta/a\sqrt{2})] \quad (19)$$

where a is the conductor radius. Even though (δ/a) may be quite large, the reactance will not greatly exceed the resistance; tuning it out is relatively easy.

Loop antennas (magnetic dipoles) have often been used for submerged reception, and at times have been proposed for transmission. It can be shown that electric and magnetic antennas with comparable dimensions have about the same radiation properties; therefore, the trailing wire, which can be made quite long, is usually superior to the loop. The impedance of a loop can be made higher by using more turns; in many cases this is a significant advantage, offsetting the greater ease with which an electric antenna can be made large.

Apparently the first analysis of a submerged loop was due to Moore,⁶ but the first published analysis in a journal was by Wait.¹¹ Moore, and later in more detail Kraichman,¹² treated a loop whose conductors are insulated, so there is no leakage current through the conducting sea. Wait considered the loop enclosed in a spherical insulating cavity; Kraichman showed that the insulating sphere has little effect on the radiation resistance provided the loop diameter is nearly as great as that of its surrounding insulating sphere. The effectiveness of a loop can be improved by loading it with high-

permeability material; Williams¹³ showed the advantage of loading with a high-permeability sphere.

The radiation resistance R_{rad} , for a loop of radius a , small compared with a wavelength in the sea, is

$$R_{rad} \approx \frac{4\omega\mu a}{3} \left(\frac{a}{\delta}\right)^2 N^2 \quad (20)$$

and the reactance x is

$$X = \omega\mu a \left[K(k) - 2 - \frac{\pi}{3}(\beta_s a)^3 + \frac{4}{15}(\beta_s a)^4 \mp \dots \right] N^2 \quad (21)$$

where $K(k)$ is an elliptic integral and N is the number of turns.

An electric dipole that is easier to match than the insulated antennas described earlier can be made by using a magnetic current loop, achieved in the form of a toroidal coil with a high-permeability core. This configuration can also be thought of as a transformer, with the current in the conducting sea being the one-turn secondary. Matching to this transformer may sometimes be easier, but, like the loop, it suffers because it cannot easily be made physically large. Considerable improvement in the performance of this antenna results from loading it with a low thin conductor.¹⁴

Arrays of submerged antennas are possible, but they do not result in increased directivity as do arrays in air. The attenuation between parts of the array, discussed previously, explains this. Why, then, should one consider arrays in conducting media? The reason is that this provides another method of solving the problem of matching low-impedance electric antennas. Because of the attenuation in the sea, antennas only a fraction of a sea wavelength apart have little interaction.

Noise and system considerations

The limiting factor in any communication system is noise. Submarine communications are subject not only to receiver noise, but to strong atmospheric noise, for low frequencies permit transmission of thunderstorm atmospherics to worldwide distances. In this section we shall compute the ranges possible for submarine communication under various conditions, emphasizing the limiting role played by atmospheric noise and by depth attenuation.

Receiver bandwidth determines the noise power that must be overcome by a communication system. Because of low carrier frequencies, the bandwidth for submarine communication is inherently limited; but an even more significant limitation is the requirement for narrow-band receivers if practical ranges are to be achieved. Thus voice bandwidths are out of the question. For many purposes the bandwidth required for noise reduction may be narrow even for code transmission. A 100-Hz bandwidth might seem minimal, but two orders of magnitude narrower are often needed for adequate noise performance, with a severely restricted communication rate.

Because noise power is proportional to bandwidth, the field expressions evaluated earlier should be combined with antenna radiation resistance figures to give ratios of transmitted to received power. The current in the transmitting antenna must be also expressed in terms of power to find this ratio.

When the insulated wire (short-circuited coaxial) an-

tenna is used, the radiation resistance is given by Eq. (18). This allows the current to be expressed in terms of the power transmitted as

$$I = 2 \sqrt{\frac{2W_t}{\omega \mu l_t}}$$

The resulting expression for the electric field above the surface in terms of transmitter power is obtained by substituting this value in (10):

$$E_a = \frac{120}{gr} \sqrt{2\sigma l_t W_t} e^{-d_t/\delta} F \cos \phi \quad (22)$$

Here we have assumed $1/\rho \ll 1$. The field in the sea at a distance is just this surface field multiplied by

$$\frac{1}{\sqrt{g}} e^{-d_r/\delta}$$

the product of refraction and depth attenuation factors.

Our example to illustrate magnitudes involved in the submarine communication problem is for

Conductivity: 4 mhos/m

Power transmitted: 10 kW

Antenna length: 50 meters

Antenna depths: multiples of 2.5 meters

Receiver bandwidth: 1 Hz

Receiver noise figure: ideal

Frequency: 10 kHz

Using these figures, the field at the surface 20 km from the submerged antenna (depth 5 meters) is $0.225 \mu\text{V/m}$ —not a very large value for a 10-kW transmitter at such a short distance. The field at a receiving antenna also submerged 5 meters is only $1.13 \times 10^{-11} \text{ V/m}$.

Determining the ratio of received to transmitted power for a receiving antenna in air requires knowledge of the antenna's impedance. The actual impedance of a vertical electric antenna in air is strongly influenced by ground properties in its vicinity. For our example, let us use the expression for the ideal short monopole above a perfectly conducting ground:

$$R_r = 40\pi^2 l_r^2 / \lambda^2 \quad (23)$$

Substituting this in (22), the power ratio between a submerged transmitter and a receiver in air is found to be

$$W_r = \frac{V^2}{4R_r} = \frac{(E_a l_r / 2)^2}{4R_r} \quad (24)$$

$$\text{or } W_r = \frac{W_t l_t}{80\pi^2 \sigma r^2} e^{-2d_t/\delta} F^2 \cos^2 \phi$$

assuming that the receiver is matched to the receiving antenna. Note the interesting result that the length of the receiving antenna does not matter. Of course, that length determines the reactance that must be tuned and the low resistance that must be matched, so it really does matter!

Using (24) in the example we calculate that the received power is 7.2×10^{-9} watt. The noise in a 1-Hz bandwidth in an ideal receiver is 0.4×10^{-20} watt; the signal-to-noise ratio is 1.8×10^{12} . If only internal receiver noise were important, it is obvious that the submarine-to-surface path could be much longer than the 20 km chosen for the example. For instance, at a 100-km distance, the signal-to-noise ratio is still 7.2×10^{10} . Beyond that distance, one should use appropriate long-distance VLF propagation theory to describe attenuation, for the iono-

sphere can no longer be neglected.

The ratio of received to transmitted power for two submerged horizontal trailing wires may be calculated from (12) and (18) to be

$$\frac{W_r}{W_t} = \frac{(El_r)^2}{4l_t^2 R_r R_t} = \frac{4l_r l_t}{\pi^2 g^2 r^2} F^2 \cos^2 \phi \\ \times \left[1 - \frac{j}{F\rho} (2F - 1) - \frac{2}{\rho^2} \right]^2 e^{-2(d_r + d_t)/\delta} \quad (25)$$

Using Eq. (25) and the parameters of the previously quoted example, except for zero depth, the signal-to-noise ratio is 1.25×10^5 . When the sum of the depths of transmitting and receiving antennas is 5 meters, the ratio becomes 2280; when the sum is 10 meters, the ratio goes down to only 42.

If only thermal noise were important, communication would be much easier than in practice, for atmospheric noise is a more important factor influencing communication range than thermal noise. This is true because static from lightning discharges has a spectrum with its most important components in the range of frequencies also suitable for submarine communication. Also, propagation at these frequencies permits signals from lightning discharges to be propagated throughout the world.

Atmospheric noise has been measured at many points on earth by numerous observers. For the frequency range down to 10 kHz, the data have been compiled by an International Working Group of the CCIR (International Radio Consultative Committee) in the form of a pamphlet.¹⁵ World maps with isonoise-field lines at 1 MHz are presented for four-hour time blocks throughout the day for each of four seasons, along with information allowing conversion of the 1-MHz information mapped for other frequencies.

At 1 MHz, diurnal variations as much as 50 dB are found at some stations, but at 10 kHz neither diurnal nor seasonal variations are very large, with extremes of about 12 dB for the total range. At frequencies below a few hundred kilohertz, noise level increases strongly with decreasing frequency. In fact, noise power varies from about $1/f^2$ to $1/f^6$ in the low-frequency range; a severe penalty is paid for going to low frequencies for tasks for which higher frequencies are suitable.

Because noise increases strongly with decreasing frequency and depth attenuation increases strongly with increasing frequency, clearly an optimum frequency exists for which the two effects balance out. Owing to the variability of noise, this optimum is not always the same even at a given location, and varies from location to location.

Let the signal-to-noise ratio in the receiver be S ; then

$$S = W_r / N \quad (26)$$

If N is given in terms of its frequency variation by

$$N = N_0 / f^n \quad (27)$$

the frequency variation for S/W_t for two submerged terminals may be found using (25):

$$S/W_t \propto f^{n+2} e^{-2d/\delta} \quad (28)$$

Differentiating with respect to frequency and setting the derivative equal to zero yields the optimum frequency condition:

$$n + 2 = d/\delta \quad (29)$$

That is, the optimum frequency makes the skin depth just equal the sum of the antenna depths divided by $(n + 2)$.

Communicating from beneath the surface to the surface requires that the signal from the submerged source must at least equal the noise level from atmospheric noise above the surface. Communication from one submerged terminal to another requires that the same condition be met. Owing to the submerged transmitter, the signal at the surface is refracted; the signal-to-noise ratio above the surface must be adequate if it is to be adequate at a submerged receiver.

These effects are best illustrated by examples, and several are presented in the ensuing discussion. Consider first the same situation in the preceding example: a field of $0.225 \mu\text{V/m}$ at a distance of 20 km from the transmitter. If this occurs during the 16–20-hour GMT time block at a point in mid-Pacific at 180° longitude and 40° north latitude during the spring, the noise voltage is $5.3 \mu\text{V/m}$ for the 1-Hz bandwidth postulated. This means that the signal-to-atmospheric-noise ratio is -27.4 dB , hardly adequate for communication. If only thermal noise had been considered, the signal-to-noise ratio would have been $+122.5 \text{ dB}$, which is quite a difference.

Increasing the transmitter power to 50 kW and doubling the antenna length to 100 meters only raises the signal-to-atmospheric noise ratio S to -17.4 dB , still inadequate. If the transmitter is at a depth of only 2.5 meters, the signal is raised another 8.7 dB, no appreciable improvement. In fact, the noise power is so great that the signal-to-noise ratio is less than unity even if there is no depth attenuation, but only refraction loss at the surface. The noise is so intense that an antenna 20 meters high has a noise voltage of 0.5 mV for 100-Hz bandwidth.

Obviously, other factors must be considered. Actually, for this example, the signal-to-noise ratio can be improved by going either up or down in frequency. Going up in frequency reduces noise, as indicated in the CCIR report. Going down to about 3 kHz reduces noise because of increased absorption there.¹⁶ The noise at 2 to 3 kHz, however, is only of the order of 10–20 dB lower than at 10 kHz; only larger reductions possible by increasing the frequency are considered in the remaining examples. For the examples that follow,

Range: 20 km

Power transmitted: 50 kW

Antenna length: 100 meters

Antenna depth: 2.5 meters and 5 meters

Receiver bandwidth: 1 Hz

Receiver noise figure: ideal

The first example is for the same mid-Pacific location (180° , 40°N) for the 12–16-hour GMT winter period. The technique for finding optimum frequency gives, in accordance with (29), $d/\delta = 4.9$, which corresponds to a frequency of 63 kHz for a depth of 5 meters. At 60 kHz the noise voltage is down to 23 dB below a microvolt, and the signal voltage is only 18.6 dB below a microvolt; the signal-to-atmospheric-noise ratio is 4.4 dB.

The effect of a lower frequency is shown by a 20-kHz example, for which the noise voltage is up to $0.95 \mu\text{V}$ and the signal is only up to $0.63 \mu\text{V}$; S is -3.5 dB , an unsatisfactory level.

On the other hand, if the antenna depth is 2.5 meters instead of 5, the 60-kHz signal is increased by 20.7 dB; S at 60 kHz is 24.4 dB, making it possible to contemplate increased bandwidth, or power and antenna reduction.

With the 2.5-meter depth at 20 kHz, the improvement is not too great, but sufficient to raise S to $+8.7 \text{ dB}$, an acceptable value.

The second example is for an area adjacent to Alaska (165°W , 65°N) for the 12–16-hour GMT block in summer. Here the optimum frequency is (for 5 meters) almost 135 kHz. At 60 kHz the noise is 15 dB below a microvolt; S is -3.6 dB . For a 2.5-meter depth, however, the 60-kHz value for S is $+17.1 \text{ dB}$.

A third example is for the same Alaskan location, but for the 16–20 time block in winter. In this case, the optimum frequency is 45 kHz, and S is $+4.5 \text{ dB}$ for a 5-meter depth. For a 2.5-meter depth, this increases to 22.9 dB.

Because the signal from the submerged transmitter must at least equal the noise above the surface where a submerged receiver is located, and further, the signal from a surface transmitter must also at least equal this noise, communication that is possible at all is feasible at least to a depth where the noise signal received from atmospheric equals the internal receiver noise. For a given frequency, therefore, we can calculate depths at which communication should be feasible for any source powerful enough to communicate by determining the depth required for attenuation and refraction losses to reduce the atmospheric noise to the receiver noise level. For the previous examples and for a unity-noise-figure receiver, these depths have been calculated as 3.06 meters at 60 kHz in mid-Pacific winter 12–16 GMT, and 7.1 meters at 45 kHz for Alaska winter 16–20 GMT. Of course, communication may be possible to a greater depth if the signal at the surface is much higher than the atmospheric noise there.

Common fallacies

Several fallacies appear from time to time in discussions of radio communication in the sea. Although some of these have been treated in this article where they have arisen, it is well to reiterate their fallacious nature.

Fallacy 1: Seawater is a dielectric at radio frequencies.

Discussion: The transition between dielectric and conductor occurs where conduction and displacement currents are equal. For seawater with conductivity of 4 mhos per meter, this occurs at 890 MHz; for all practical communication purposes, the sea is a good conductor.

Fallacy 2: Atmospheric noise can be neglected in radio communication through the sea.

Discussion: The preceding section has shown that atmospheric noise profoundly affects communication ranges and frequencies in the sea. The optimum frequency in the absence of atmospheric noise effects would be obtained by using Eq. (29) with $n = 0$ —and it would be much lower than that found when noise is taken into account. Furthermore, noise is the limiting factor that requires large power for receiving antennas near the surface, rather than depth attenuation and refraction. Thus both surface and submerged transmitters must be much larger because of atmospheric noise than they would need to be otherwise. Also, atmospheric noise requires submerged transmitting antennas to be near the surface and large, whereas (25) would indicate that depth and length of transmitting and receiving antennas for submerged-to-submerged communication have the same effect.

Fallacy 3: The tangential component of E in air, and thus the downgoing E in water, can be increased by

transmitting vertically downward with a high-altitude horizontal antenna.

Discussion: The tangential component of the field was shown to be the same for downcoming and horizontally traveling waves, because the reflection coefficient for downcoming waves is so near unity that most of the incident field is canceled by the reflected field in air.

Fallacy 4: Because a dc pulse propagates with less attenuation than an exponential pulse, it should be superior for communication in the sea.

Discussion: Propagation of electromagnetic waves in a conductor follows the same equation as heat conduction and diffusion. The highest point of a pulse that satisfies the diffusion equation is indeed attenuated more slowly than an exponential; the reason is that at longer distances the peak is due to lower-frequency components of the original pulse. Fourier analysis of the transmitted pulse shows frequency components over a wide range. The higher frequencies are attenuated in shorter distances; at longer distance the by-that-time sloppy pulse is primarily due to the lower of the original frequencies. Thus energy that went into the higher-frequency components has been wasted, and it would have been better not to transmit them at all.¹⁷

Fallacy 5: A large-loop antenna that is small compared with a long wavelength in air is also small compared with a wavelength in sea.

Discussion: The wavelength in sea is, as has been shown, much shorter than in air. A loop antenna the size of a submarine would therefore seem small compared with a wavelength in air, and could be treated there as a magnetic dipole. In the sea, on the other hand, such a loop is likely to be many wavelengths on a side; furthermore, attenuation along the antenna and between its top and bottom tend to make it appear more like an electric dipole located near its shallowest point.

Conclusions

Radio communication from, to, and between submerged antennas involves propagation paths through the highly conducting sea with segments vertically above the submerged antennas and a segment along the surface like that for a vertical antenna in air. In fact, a submerged horizontal electric dipole is equivalent in its fields to a weaker vertical quadrupole at the surface. Because of the strong attenuation in the sea, very low frequencies are required for such communication.

Antennas are quite different in the sea, owing to the effects of high conductivity. Both insulated wire electric antennas and loop magnetic antennas can be used successfully, but the radiation resistances are quite low and matching problems often exist.

Communication ranges are limited both by the effects of depth attenuation and by atmospheric noise. Because of these factors, communication from submerged transmitters is necessarily limited to rather short ranges, even when antennas are close to the surface. Ranges in the tens of kilometers are possible for antennas within 5 meters of the surface if bandwidths are restricted to the order of 1 Hz, transmitting trailing-wire antennas are used, and powers of tens of kilowatts are available. For communication from surface transmitters to submarines, much longer ranges are possible, requiring very large transmitters to overcome atmospheric noise. Because of atmospheric and attenuation, carrier frequencies most

useful are in the tens of kilohertz range.

Communication with submerged terminals is possible provided sufficiently large power is coupled with sufficiently low communication rates. Because the ranges are restricted by fundamental attenuation considerations and by noise that must be considered an unchangeable part of the environment, significant breakthroughs are not to be expected in submarine radio communication. Nevertheless, both submarines and those exploring and exploiting the ocean bottom can use radio for essential communications; for two-way communication the transmitting antenna will ordinarily be quite close to the surface.

Direct communication between surface vessels and ocean-bottom working parties could, if need be, use extremely low-frequency carriers with the surface ship also using a submerged antenna if the path is sufficiently short that direct attenuation from one antenna to the other is not prohibitive. The submerged transmitting antenna from the surface ship would therefore only have to compete with atmospheric noise that had undergone refraction loss of the order of 60 dB and its own signal would not have to experience that loss; the power required would therefore be reduced accordingly. Presumably such communication would have to be at frequencies well below 10 kHz to overcome direct transmission loss. In most cases underwater sound appears superior for this application.

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Disaster control coordination for large interconnected systems

Since widespread power failures have proved to be more than a remote possibility, new approaches to interconnection problems and coordination of disaster control procedures have evolved

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A new philosophy for the analysis and design of large electric power interconnections has evolved following the Northeast power failure of November 9, 1965. Most contingencies that can cause widespread outages occur too infrequently to be included in the criteria for system design. Nevertheless, systems should be tested for combinations of events that cause system instability and separation so that the consequences of these unlikely occurrences can be evaluated and system designers can provide disaster control procedures to limit the extent and duration of system outages. The design and disaster control procedures should provide successive lines of defense against increasingly severe and unlikely events. Disaster control procedures can be coordinated so that governor action, load shedding, and system separation can be integrated according to time and frequency in such a way that maximum reliability and security will be provided.

Most of the power systems in the United States and Canada are part of the same interconnected system. More than 200 000 000 kW of generating capacity operates in synchronism, and the stored energy and spinning reserve of this network will replace large amounts of lost generation with hardly noticeable frequency deviations. If a generator anywhere in this interconnection trips off the line, its loss is immediately supplied from the remainder of the interconnection until the area in which the loss has occurred is able to increase generation to restore its tie lines to schedule.

Widespread outage in this large interconnection is invariably accompanied by separation of the affected area from the network. Traditionally, system design criteria have included design contingencies that the system must

withstand without loss of load or damage to equipment. These contingencies include such events as permanent three-phase faults or loss of the most heavily loaded line. Other more severe combinations of contingencies may be included as design criteria, but there is a practical limit to the amount of money that can be spent to prevent system separation due to extremely unlikely events. Nevertheless, it is proposed that system designs be tested for these unlikely events that cause system separation so that the power system designer can specify and coordinate disaster control procedures to limit the extent and duration of system outages. Specific studies of system response to loss of generation will aid in the selection of critical frequencies to be considered in the coordination of these disaster control procedures.

System behavior during abnormal conditions

A study of the Northeast interconnection following the power failure of November 9, 1965, shows that any part of the interconnected system can be isolated by severe disturbances and left with rapidly decreasing frequency.¹ The behavior of certain essential elements of a power system during abnormal conditions of frequency and voltage will control the selection of basic parameters of frequency and time in the coordination of load shedding, and system separation.

Governor response. Following a sudden unbalance in load and generation, governor response is very effective in arresting the initial decay of frequency. The initial rate of change of frequency will be a function of the excess load on the system and the inertia of its generators and can be expressed as

$$R = \frac{60}{2H} \frac{L - G}{G} \quad (1)$$