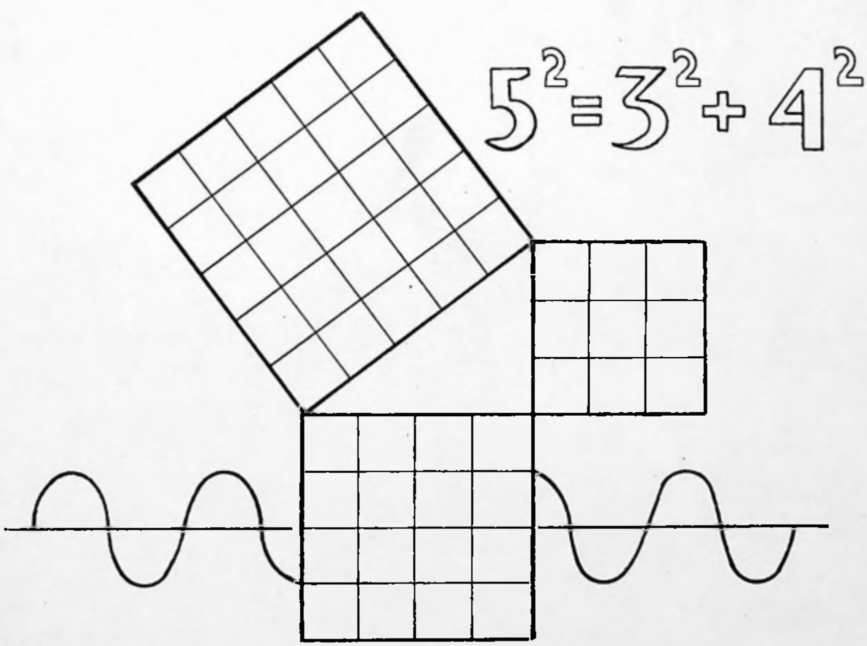
ARMY EXTENSION COURSES

United States Army Signal Center

and School

PRO PATRIA VIGILANS

MATHEMATICS



SUBCOURSE 100

22 MARCH 1963

(Rev 9 Jan 64)

YOUR ASSISTANCE!

PLEASE READ THE FOLLOWING INFORMATION BEFORE YOU BEGIN YOUR STUDY. IF YOU FOLLOW THE INSTRUCTIONS GIVEN BELOW, YOU WILL SIMPLIFY OUR WORK HERE AT THE US ARMY SIGNAL SCHOOL AND THUS CONTRIBUTE TO YOUR OWN PROGRESS IN EXTENSION COURSE STUDY.

YOUR OBLIGATIONS - - -

Do not write in or deface subcourse textbooks.

Return these materials in accordance with specific instructions as outlined in this subcourse.

Always print your name and address clearly.

Do not write comments or questions on the lesson answer sheets. Put them on a separate sheet of paper, with your name, subcourse and lesson.

Notify us immediately of any change in address or status, using the form on the following page.

Do not divulge the solutions to test questions. The completed work that you submit must be your own.

Allow for processing and mailing time—10 days to 2 weeks—for our replies and return of your answer sheets.

YOUR WORK SPEED - - -

Complete this subcourse as quickly as possible. If this subcourse is revised, replaced, or withdrawn while you are studying it, you will have 8 months to complete it.

YOUR SUBCOURSE GRADE - - -

Your subcourse grade is the one that you earn on the examination.

Detach and Mail

CHANGE IN STATUS

If you have recently had a change of address or some other change in status, please detach this sheet, complete the form below, and return it with your next lesson. The completed form should include both new information and that which has not changed. PLEASE PRINT.

NAME: ____
(Last) (First) (Initial)

RANK: ____________________ SERVICE NO.: ____________________

ADDRESS: ____

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UNIT TO WHICH ASSIGNED AND ADDRESS:

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RETIREMENT ANNIVERSARY DATE: ____________________

OTHER CHANGE: ____
(Describe Change) ____

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SUBCOURSE 100, MATHEMATICS

INTRODUCTION

If you are to begin a study of electronics it is first necessary to become familiar with the language of number and size—mathematics. In the first place, you will find that a practical knowledge of mathematics is required for the solution of problems in the fields of telephony, radio, radar, television, photography, meteorology, and automatic data processing. In the second place, an understanding of the relationships between the various electrical quantities is necessary in grasping the most basic theories of electronics. It is with this dual purpose in mind that we have written this subcourse.

Obviously, for students who have already studied mathematics, this short course can serve only as a refresher. Those studying the subject for the first time may find the material a little more demanding of their time.

This subcourse consists of seven lessons and an examination, as follows:

Lesson 1. Introduction to Algebra
Lesson 2. Algebra
Lesson 3. Logarithms
Lesson 4. Geometry
Lesson 5. Trigonometry—I
Lesson 6. Trigonometry—II
Lesson 7. Functions and Graphs
Examination
Credit Hours: 17

You are urged to finish this subcourse without delay; however, there is no specific limitation on the time you may spend on any lesson or the examination. For statistical purposes, you are requested to enter on each answer sheet the total number of hours spent on the solution.

Texts and materials furnished:

TM 11–684, Principles and Applications of Mathematics for Communications-Electronics, October 1961
Attached Memorandum

You may keep the Attached Memorandum. However, all other textual material will be returned to the school when you have been notified that you have *successfully completed* the subcourse. Please do not deface the texts, as they are to be used by other students.

LESSON 1

INTRODUCTION TO ALGEBRA

CREDIT HOURS - - - -	2
TEXT ASSIGNMENT - - -	TM 11–684, par 1–42; Attached Memorandum, par 1–4
MATERIALS REQUIRED - -	None
LESSON OBJECTIVE - - -	Review of exact and approximate numbers, percentage, powers and roots; introduction to algebra, using negative and positive numbers
SUGGESTIONS - - - -	1. The following procedure for studying lessons of this subcourse is recommended: a. Read the lesson assignment and carefully study the solutions of the examples. b. Solve a few of the review problems at the end of each lesson. The answers to these problems will be found on page 232 of the text. c. Solve the problems in the lesson booklet. 2. Conversion factors may be found in paragraph 3 of appendix I in TM 11–684. These are used to convert one unit of measurement to another.

ATTACHED MEMORANDUM

(This attached memorandum has been prepared under the supervision of the Commandant, US Army Signal School. It is provided for resident and nonresident instruction conducted by the US Army Signal School. It reflects the current thought of this school and conforms to printed Department of the Army doctrine as closely as possible. Development and progress render such doctrine continuously subject to change.)

1. EXACT AND APPROXIMATE NUMBERS

In a discussion of methods of calculation, two types of numbers are recognized—exact and approximate numbers.

a. An *exact* number may be considered to be the result of counting. The number of trucks in a motor pool (38) or the amount of money in a soldier's pocket after payday ($116.23), can be expressed precisely by careful counting. If a glass of beer costs 15 cents, the total cost of 6 beers, 90 cents, can be determined exactly by ordinary multiplication, $6 \times 15 = 90$.

b. An *approximate* number is one that arises as a result of measurement or estimation. Suppose that the distance between two points is carefully measured to the nearest hundredth of a foot on an accurate steel tape and recorded as 231.47 feet. This means that the true distance is not greater than 231.475 feet and not less than 231.465 feet, because any length slightly smaller than 231.465 would have been recorded as 231.46 feet and any length slightly greater than 231.475 would have been recorded as 231.48 feet. The recorded distance of 231.47 feet has a maximum possible error, or deviation from the true value, of 0.005 feet. As another illustration, suppose that the value of π is written as 3.14. This is also an approximate number since π may be written more precisely as 3.1416. The value of π given first has an error or deviation of about 0.0016, the latter about 0.00001. An approximate number correctly written in decimal form has at most an error of 5 in the decimal place immediately following the last digit. This decimal place may be called the decimal place of error.

c. The number of significant digits in an approximate number expressed as a decimal is the number of digits obtained by counting from left to right beginning with the first non-zero digit and ending with the digit just preceding the decimal place of error. Thus, 231.47 feet, the distance between 2 points, is written with 5 significant digits: 3.14, the value of π, has 3 significant digits; 0.040, the diameter of a wire, has 2 significant digits; 100.0 feet, the length of a table, has 4 significant digits; and 34, the number of trucks in a motor pool, has an unlimited number of significant digits.

2. THE ADDITION AND SUBTRACTION OF APPROXIMATE NUMBERS: PRECISION

a. Assume that the following numbers are to be added.

$$\begin{array}{r} 13.698 \\ 5.4 \\ 287.27 \\ 52.49 \\ \hline \end{array}$$

It would be incorrect to add these numbers as they now stand and write 358.858 as the sum. The reason is that we would be treating the blank spaces following 5.4, 287.27, and 52.49 as if they were zeros. These blank spaces are not zeros, but unknown digits. Placing zeros in these spaces would indicate an accuracy that is not justified by the method of measurement. A more useful and correct way to add is to round off to the first decimal place before adding, as shown below.

$$\begin{array}{r} 13.7 \\ 5.4 \\ 287.3 \\ 52.5 \\ \hline 358.9 \end{array}$$

The sum, 358.9, is a much better answer than the one given above. An even more reliable answer is obtained by rounding off the sum to 359, because an accumulation of errors due to rounding off might render the tenths decimal place inaccurate.

b. The concept of *precision* is appropriate to addition and subtraction for it is closely related to the idea of error. We think of a number as being precise (not accurate) to a given unit which in many cases means precise to a given decimal place. For example, hundredths of a foot, thousandths of an inch, tenths of a second, and dollars. We add or subtract numbers that are precise to the same unit. That is, in addition and subtraction of numbers expressed as decimals, the numbers must be expressed to the same decimal place as shown in paragraph 2*a*. These facts may be summarized in the following rule: *The sum or remainder obtained from the addition or the subtraction of approximate numbers should have no greater precision than that of the least precise number involved in these operations.*

c. It is well to remember that in practice a number is always rounded off to the nearest useful value. Thus, 23.458 may be rounded off to 23.46 or to 23.5; 56.521 is rounded off to 56.5. It is usually considered good practice to round off to the nearest *even* digit if the digit to be dropped is exactly 5. Thus, 53.315 rounds off to 53.32, but 73.265 rounds off to 73.26. However, 73.2652 rounds off to 73.27.

3. THE MULTIPLICATION AND DIVISION OF APPROXIMATE NUMBERS: ACCURACY

a. The ordinary method of multiplying two approximate numbers is shown below.

$$\begin{array}{r} 728.18 \\ 32.6 \\ \hline 436.908 \\ 1456.36 \\ 21845.4 \\ \hline 23738.668 \end{array}$$

To express the product as 23,738.668 is to imply an accuracy in the answer which cannot be justified. For example, since 32.6 is an approximate number, it may represent any value between 32.55 and 32.64. Since 728.18 × 32.55 = 23,700 and 728.18 × 32.64 = 23,770, it can be seen that only 3 significant digits can be saved in this product. Thus, the answer should be written 23,700.

b. The concept of *accuracy* is suited to multiplication and division because it is related to the notion of *relative error. The product or quotient obtained from the multiplication or the division of approximate numbers should contain no more significant digits than occur in the least accurate factor.*

4. FRACTIONAL EXPONENTS

a. These can be illustrated by expressing the examples found in paragraph 23, TM 11–684, somewhat differently, as follows:

$$\sqrt{25} = (25)^{\frac{1}{2}} = 5$$

$$\sqrt[3]{216} = (216)^{\frac{1}{3}} = 6$$

$$\sqrt[4]{81} = (81)^{\frac{1}{4}} = 3$$

These are fractional exponents.

b. Fractions may also be used as exponents to indicate that the number is to be raised to a power and a root taken. For example,

$$5^{\frac{3}{2}} = \sqrt{5^3} = \sqrt{125}$$

$$3^{\frac{2}{3}} = \sqrt[3]{3^2} = \sqrt[3]{9}$$

EXERCISES

In each of the following exercises, select the ONE answer that BEST completes the statement or answers the question. Indicate your solution on the answer sheet by filling in with pencil the appropriate lettered block.

All exercises of this lesson are of equal weight. The total weight is 100.

1. A certain 47,000-ohm resistor is said to have a 5% tolerance. That is, its value must be within 5% of the nominal value. The maximum value that can be accepted within the tolerance is

a. 47,000 ohms.
b. 47,235 ohms.
c. 49,350 ohms.
d. 52,000 ohms.

2. If an electric motor with an output power of 248 watts requires an input power of 277 watts, what percent (efficiency) of the input power is the output power?

a. 85.5%
b. 89.5%
c. 95.5%
d. 111.7%

3. The following measurements (approximate numbers) have been made: 26.95 in, 0.027 in, 7.2 in, and 0.0042 in. The correct expression of their sum is

a. 34.1 in.
b. 34.2 in.
c. 34.18 in.
d. 34.034 in.

STRIP IN ON PAGE 4

4. The resistance of wire is directly proportional to its length. If a 1,000-foot section of wire has a resistance of 0.248 ohm, the resistance of a 2,500-foot section is

a. .372 ohm.
b. .620 ohm.
c. 6.20 ohms.
d. 620 ohms.

5. The weight of 21-gage wire is directly proportional to the length. If a 1,000-foot section weighs 2.452 pounds, a 1,250-foot section weighs

a. 2.452 pounds.
b. 3.065 pounds.
c. 30.65 pounds.
d. 306.5 pounds.

6. When the impedance of an ac circuit is to be computed, the problem is normally reduced to finding the square root of a number. If the problem is reduced to finding the square root of 273, the impedance is

a. 16.5 ohms.
b. 34.1 ohms.
c. 50.6 ohms.
d. 68.3 ohms.

7. Assume that you must determine the resonant frequency of a parallel circuit and it is necessary to find the square root of .02584. The square root of this number is

a. .016.
b. .051.
c. .161.
d. .508.

8. The weight (W, in grams) of a cube of brass is given by the formula $W = 8.4X^3$, where X is the length in centimeters of each side. If $X = 6.4$ cm, the weight of the cube is approximately

a. 160 grams.
b. 340 grams.
c. 1,560 grams.
d. 2,200 grams.

9. The voltage (E) required to produce a given amount of power across a resistor may be computed from the formula $E = \sqrt{PR}$. If the power (P) is 4.0 watts and the resistance (R) is 1,200 ohms, the required voltage is approximately

a. 14 volts.
b. 22 volts.
c. 44 volts.
d. 69 volts.

10. The maximum range of communication between two VHF radio relay stations can be determined from the formula $d = 2.46\sqrt{h}$ where d is the range (miles) and h is the height (feet) of the antennas (both antennas are of the same height). If the antennas are 45 feet high, the maximum distance between them is approximately

a. 10 miles.
b. 12 miles.
c. 16 miles.
d. 23 miles.

11. The impedance (Z) of a circuit containing resistive and reactive components is expressed by the formula $Z = \sqrt{R^2 + X^2}$; R is the resistance and X is the reactance. If $R = 35$ ohms and $X = 58$ ohms, the total impedance of the circuit is

a. 14 ohms.
b. 21 ohms.
c. 68 ohms.
d. 92 ohms.

12. Assume that you are required to requisition and supervise the installation of a cylindrical water tank like that shown in figure 1. The volume (V) of the tank must be 40,000 cubic feet. If

the height (h) is 16.0 feet, the diameter (d) of the circular base as determined from the formula $d = \sqrt{\frac{1.27V}{h}}$ is approximately

a. 17.8 ft.
b. 56.3 ft.
c. 178 ft.
d. 563 ft.

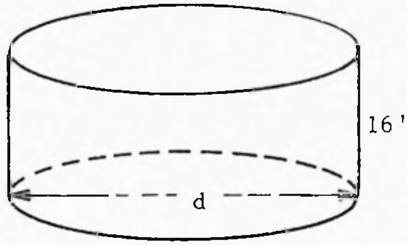

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Figure 1. Cylindrical water tank.

13. If individual voltages of +90v, +75v, +15v, −20v, and −5v are connected in series, the total voltage (algebraic sum) is

a. +125 volts.
b. +155 volts.
c. +165 volts.
d. +205 volts.

14. Two angles can be added by finding their algebraic sum in degrees. Thus, the sum of a +240-degree angle and a −115-degree angle is

a. −125 degrees.
b. +125 degrees.
c. −355 degrees.
d. +355 degrees.

15. At an electronic equipment testing station in Antarctica, the thermometer reading at 0200 hours was −37 degrees Fahrenheit. Experience with that thermometer has shown that the true temperature must be computed by subtracting a correction factor of +3 degrees from the thermometer reading. The true temperature was

a. −34 degrees.
b. +34 degrees.
c. −40 degrees.
d. +40 degrees.

16. The total voltage in a circuit is the sum of the individual voltages. A circuit with voltages of −109.4v, +118.8v, +130.2v, and −139.6v has a total voltage of

a. zero.
b. −8.8 volts.
c. +18.8 volts.
d. −41.6 volts.

17. The value of voltage in an ac circuit at any given instant of time is equal to the product of the maximum voltage and an algebraic expression having a value of −0.866. If the maximum voltage is 156 volts, the instantaneous voltage is approximately

a. −135 volts.
b. +135 volts.
c. −181 volts.
d. +181 volts.

18. Assume that while collecting data at an arctic weather station you find a Fahrenheit (F) thermometer is available and the data must be presented in degrees centigrade (C). The formula used

for conversion is $C = \frac{5}{9}(F - 32)$. If the temperature is $-40°F$, the centigrade reading will be

a. 5°C.
b. −16°C.
c. −40°C.
d. −148°C.

19. Minus and plus signs are often used in electrical work to indicate the direction of current flow or voltage level with respect to a reference point. If the current (in amperes) can be determined by dividing the voltage (−108 volts) by the resistance (1,200 ohms), the current will have a value of

a. −90 milliamperes.
b. +90 milliamperes.
c. −110 milliamperes.
d. +110 milliamperes.

HINT: Consider the resistance to have a positive value.

20. Assume that a +300-volt potential is applied across resistance A and battery B shown in figure 2. Since the sum of voltage A and voltage B (−150 volts) must equal +300 volts, what is the voltage A?

a. −150 volts
b. +150 volts
c. +300 volts
d. +450 volts

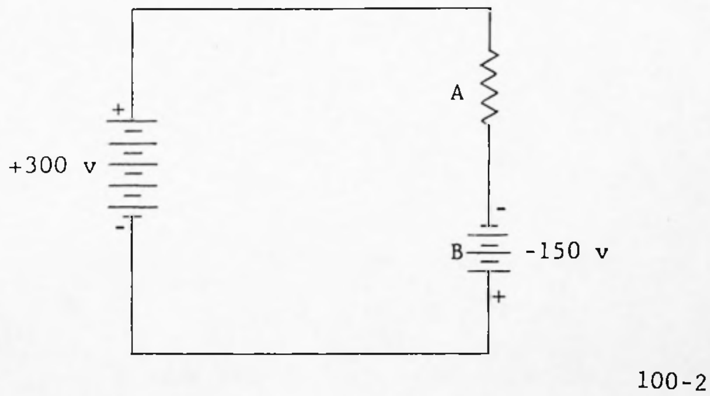


Figure 2. Series voltage circuit.

LESSON 2

ALGEBRA

CREDIT HOURS - - - - - 2

TEXT ASSIGNMENT - - - TM 11–684, par 43–50; 62–76; 80–86

MATERIALS REQUIRED - - None

LESSON OBJECTIVE - - - To familiarize you with the fundamental operations of algebra and the solution of equations and formulas

SUGGESTIONS - - - - When studying paragraphs 44–51, you should keep in mind that the main emphasis of this lesson is on the solution of equations and formulas.

EXERCISES

In each of the following exercises, select the ONE answer that BEST completes the statement or answers the question. Indicate your solution on the answer sheet by filling in with pencil the appropriate lettered block.

All exercises of this lesson are of equal weight. The total weight is 100.

21. At an altitude *a* (in feet) above sea level, the distance *d* (in miles) that a person can see an object is given by the formula $d = \left(\frac{3a}{2}\right)^{\frac{1}{2}}$. If you are observing a surface target located 8 miles at sea, you must be located above sea level at least

a. $3\frac{1}{2}$ feet. *b.* $10\frac{2}{3}$ feet. *c.* $42\frac{2}{3}$ feet. *d.* 144 feet.

22. The energy converted into heat in an electric heater may be expressed by the formula $H = I^2Rt$. If $I = \frac{E}{R}$, this formula may be rewritten as

a. $H = Et$. *b.* $H = ERt$. *c.* $H = \frac{E^2t}{R}$. *d.* $H = I^2Et$.

23. The formula for centripetal force may be written in the form $F = \frac{W(gt)^2}{gr}$. This formula can be simplified to

a. $F = \frac{Wgt^2}{r}$. *b.* $F = \frac{Wt^2}{r}$. *c.* $F = \frac{Wg^2t^2}{gr}$. *d.* $F = Wgt$.

24. Mechanical power may be computed from the formula $P = \left(\frac{2s}{t}\right)m \cdot at$. When simplified, this formula becomes

a. $P = 2ams$. *b.* $P = 2amst^2$. *c.* $P = amst$. *d.* $P = 2ms$.

25. The formula for computing the total resistance of three parallel resistors is $\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$. An equivalent formula that gives another method of computing the total resistance is

a. $R_T = R_1 + R_2 + R_3$.

b. $R_T = \frac{R_1 + R_2 + R_3}{R_1R_2R_3}$.

c. $R_T = \frac{R_1R_2R_3}{R_1 + R_2 + R_3}$.

d. $R_T = \frac{R_1R_2R_3}{R_1R_2 + R_1R_3 + R_2R_3}$.

26. The power (P) lost in an electrical circuit is equal to the square of the current (I) multiplied by the resistance (R). The formula for the current in terms of the power and resistance is written as

a. $I = \sqrt{PR}$.

b. $I = PR$.

c. $I = \sqrt{\dfrac{P}{R}}$.

d. $I = \dfrac{P^2}{R^2}$.

27. The inductive reactance (X_L) in a circuit can be determined by computing the product of 2π, frequency (f), and the inductance (L). The inductance for a given reactance and frequency can be computed by the formula

a. $L = 2\pi f X_L$.

b. $L = \dfrac{2\pi X_L}{f}$.

c. $L = \dfrac{X_L}{2\pi f}$.

d. $L = \dfrac{f}{2\pi X_L}$.

28. If the formula used to convert Fahrenheit temperatures to centigrade is $C = 5/9\ (F - 32)$, the formula used to convert centrigrade readings to Fahrenheit is

a. $F = 9/5C + 32$.

b. $F = 9/5(C + 32)$.

c. $F = 5/9C + 32$.

d. $F = 5/9(C + 32)$.

29. Given that $I = \dfrac{CE}{t}$ amperes and $Q = It$ coulombs. Another formula for finding Q, which combines these two, is the one that may be written as

a. $Q = \dfrac{C}{E}$.

b. $Q = \dfrac{CE}{t^2}$

c. $Q = CEt$.

d. $Q = CE$.

30. Given that $W = \dfrac{QV}{2}$ and $C = \dfrac{Q}{V}$. Another formula for finding W, which combines these two, is the one that may be written as

a. $W = \dfrac{C}{2}$.

b. $W = \dfrac{CQ}{2}$.

c. $W = \dfrac{2Q}{C}$.

d. $W = \dfrac{CV^2}{2}$.

31. In solving a problem concerning an electrical circuit, the following two equations were obtained: $4I_1 - 3I_2 = 100$, and $3I_1 + 5I_2 = -50$. The value of I_1 is

a. 3.8 amps.
b. 12.1 amps.
c. 17.3 amps.
d. 22.7 amps.

32. The energy (P) stored in an inductive circuit may be determined by squaring the value of current (I), multiplying it by the inductance (L), and dividing by two. If the energy is 100 joules, the inductance 80 henrys, the current will be

a. 1.3 amps.
b. 1.6 amps.
c. 2.5 amps.
d. 6.3 amps.

33. The magnetic field (H) of a solenoid may be expressed as $H = 0.4\pi NI$. What number of turns N is required to produce a magnetic field (H) of 300 oersteds when the current (I) in the circuit is 2 amperes? ($\pi = 3.14$)

a. 119 turns *b.* 188 turns *c.* 350 turns *d.* 726 turns

34. The resistance (R_t) of a copper wire at a given temperature (t) is given by the formula $R_t = R_o\ (1 + 0.0042t)$ where R_o is the resistance at 0° centigrade. If $R_o = 27$ ohms, the temperature when $R_t = 35$ ohms is approximately

a. 2 degrees. *b.* 71 degrees. *c.* 150 degrees. *d.* 300 degrees.

35. An antenna is to be installed in a field whose width is one half the length. When the length of the field is increased by 15 feet and the width kept the same, the area is increased by 450 sq ft. The original length of the field is

a. 20 feet. *b.* 30 feet. *c.* 60 feet. *d.* 75 feet.

36. If two capacitors (C_1 and C_2) are connected in series, the total capacitance (C_T) can be determined by using the formula $\frac{C_T}{C_1} + \frac{C_T}{C_2} = 1$. If $C_1 = 30$ mf and $C_2 = 40$ mf, then C_T is equal to approximately

a. 14 mf. *b.* 17 mf. *c.* 35 mf. *d.* 70 mf.

37. The internal resistance of a 12-volt battery can be determined by using the formula $I = \frac{E}{R + r}$. If the external resistance (R) is 0.5 ohm and the current (I) is 6 amperes, the internal resistance (r) is

a. 0.7 ohm. *c.* 1.5 ohms.
b. 1 ohm. *d.* 2 ohms.

38. If an ac circuit has an inductance (L) and capacitance (C), its resonant frequency (f) can be determined from the equation $2\pi fL = \frac{1}{2\pi fC}$. Solving the equation for f results in the formula

a. $f = \frac{1}{2\pi\sqrt{LC}}$. *c.* $f = \frac{2\pi(L \cdot C)}{1}$.
b. $f = \frac{1}{2\pi LC}$. *d.* $f = \frac{2\pi}{LC}$.

39. The formula for finding the force between two magnetic poles is $F = \frac{S_1S_2}{L^2}$. If a force $F = 1.5$ dynes and the strength of the two poles is $S_1 = 60$ units and $S_2 = 90$ units, then the distance L between them is

a. 0.06 cm. *c.* 60 cm.
b. 0.60 cm. *d.* 360 cm.

40. The formula for determining the number of turns on the primary winding of a radio receiver power transformer is $N = \frac{280\ E}{fA}$, where E is the primary voltage, f is the frequency, and A is the cross sectional area (square inches) of the core. If $E = 120$ volts and $f = 60$ cps, what cross sectional area will require 1120 turns?

a. 0.5 square inch *c.* 1.5 square inches
b. 1.0 square inch *d.* 2.0 square inches

LESSON 3

LOGARITHMS

CREDIT HOURS - - - - - 2

TEXT ASSIGNMENT - - - TM 11–684, par 104–127; Attached Memorandum, par 5

MATERIALS REQUIRED - - None

LESSON OBJECTIVE - - - To familiarize you with the use of scientific notation and the methods of computing arithmetic problems by using logarithms

SUGGESTIONS - - - - None

ATTACHED MEMORANDUM

5. LOGARITHMIC EQUATIONS

A logarithmic equation is one that involves one or more unknowns in a logarithm. For example, x = log 300 and log x = 2.42. Such equations are solved by logarithms. In the first example, x is determined by finding the logarithm of 300. Thus,

$$x = \log 300 = 2.4771.$$

In the second example, x is determined by finding the antilogarithm of 2.42. Thus,

$$\log x = 2.42$$
$$x = \text{antilog } 2.42 = 263.$$

EXERCISES

In each of the following exercises, select the ONE answer that BEST completes the statement or answers the question. Indicate your solution on the answer sheet by filling in with pencil the appropriate lettered block.

All exercises of this lesson are of equal weight. The total weight is 100.

41. The charge of an electron is given as 0.000000000480 esu. This numerical value may be expressed in scientific notation as

a. 480×10^{-12}.
b. 4.80×10^{-10}.
c. 4.80×10^{-9}.
d. 4.80×10^{10}.

42. The wavelength (in meters) of a radio wave is expressed by the formula $\lambda = c/f$. If c (velocity of light) is 3.00×10^{8} meters/sec, and f (frequency) is 225×10^{6} cps, the length of the wave is approximately

a. 0.8 meter.
b. 1.3 meters.
c. 7.5 meters.
d. 13.3 meters.

43. In computing the resonant frequency of an LC circuit, it is necessary to find the product of the values of inductance (L) and capacitance (C). If $L = 19 \times 10^{-6}$ henry and $C = 66 \times 10^{-6}$ farad, the product of the two is

a. 1.3×10^{3}.
b. 1.3×10^{-6}.
c. 1.3×10^{-9}.
d. 1.3×10^{-15}.

44. The capacitance of a parallel-plate paper capacitor, the area of whose plates is A (square inches) and the distance between them is d (inches), is given by the formula $C = (79 \times 10^{-14})\frac{A}{d}$. If the area is 4.5×10^2 sq inches and the distance between plates is 0.015 inch, the capacitance is

a. 0.024×10^{-6} farad.
b. 0.237×10^{-6} farad..
c. 0.024×10^{-12} farad.
d. 2.37×10^{-12} farad.

45. The noise figure (decibels) of a radar receiver is given by the formula $F_{db} = 10 \log F$. If F (noise figure ratio) is 25, the value of F_{db} is

a. 14 db.
b. 40 db.
c. 140 db.
d. 250 db.

46. The gain of an amplifier is given by the formula $db = 10 \log \frac{P_2}{P_1}$. If P_1 (input power) is 8.0×10^{-6} watts, and P_2 (output power) is 10 watts, the gain is

a. 61 db.
b. 69 db.
c. 90 db.
d. 97 db.

47. The characteristic impedance of a two-wire open-air transmission line is expressed by the formula $Z_0 = 276 \log \frac{b}{a}$. If b (distance *between* wires) is 8.00 inches and a (radius of each wire) is 0.0254 inch, the characteristic impedance is

a. 138 ohms.
b. 315 ohms.
c. 415 ohms.
d. 690 ohms.

48. The density d of silver deposit on a photographic film is defined by the relation $d = \log O$ where O is the opacity. If the transparency T is the reciprocal of the opacity, the transparency of a film with a density of 2.7 is

a. 0.002.
b. 0.005.
c. 0.007.
d. 0.009.

49. Assume that a 1-mile spiral-four cable line has a 0.78 db loss, where $Loss_{db} = 10 \log \frac{P_i}{P_o}$. If the input power P_i is 0.10 watt, the output power P_o is

a. 0.069 watt.
b. 0.084 watt.
c. 0.169 watt.
d. 1.367 watts.

50. It has been found that in one year the cost of producing 2753 airplanes of a certain type was $2,017,000,000. If logarithms are used to compute the cost of each plane, the logarithm of the cost per plane will be

a. 5.7445.
b. 5.8649.
c. 6.7445.
d. 6.8649.

HINT:

$$\text{Cost per plane} = \frac{\text{Total cost}}{\text{No. of planes}}$$

51. Wavelength (λ) is sometimes expressed as $\lambda = 1884\sqrt{LC}$ where L is the inductance in microhenrys and C is the capacitance in microfarads. If the value of LC is 0.05198, the logarithm of the wavelength is

a. 6.0828.
b. 3.9172.
c. 2.6330.
d. 2.1909.

52. The area A (acres) of a triangular piece of ground whose sides are *a, b,* and *c* rods is given by the formula $A = \dfrac{\sqrt{s(s-a)(s-b)(s-c)}}{160}$ where $s = \dfrac{(a+b+c)}{2}$. The site of a proposed signal center is a triangular area whose three sides are 33.0 rods, 39.0 rods, and 46.0 rods in length. The logarithm of the area of the site is

a. 0.5963. *b.* 1.6984. *c.* 3.3967. *d.* 5.0045.

53. The quantity of charge (coulombs) on a particular capacitor is given by $q = 0.005\left(1 - \dfrac{1}{e^{0.3}}\right)$. If e = 2.718, the charge on the capacitor (computed with logarithms) is

a. 1.30×10^{-3} coulomb.
b. 2.17×10^{-3} coulomb.
c. 3.71×10^{-3} coulomb.
d. 9.35×10^{-3} coulomb.

54. The cathode current (I_k) of a triode tube is given by $I_k = 0.0005(5 + 31.3)^{\frac{3}{2}}$. The logarithm of the cathode current is

a. 0.1094. *b.* 1.5599. *c.* 9.0389 − 10. *d.* 9.1699 − 10.

55. The Steinmetz equation for hysteresis showing the loss of energy in ergs/cycle/cubic centimeter is $W = 0.06B^{1.6}$, where B is the maximum induction in maxwells per square centimeter. If the maximum induction of a power transformer is 3.4 maxwells per square centimeter, the hysteresis loss (computed with logarithms) is

a. 0.3 erg/cycle/cm^3.
b. 0.4 erg/cycle/cm^3.
c. 0.5 erg/cycle/cm^3.
d. 0.6 erg/cycle/cm^3.

56. The db gain of an audio amplifier can be computed by using the formula $\text{db} = 10 \log \dfrac{P_2}{P_1}$. If the input signal power (P_1) is 10 milliwatts and the output signal power (P_2) is 120 milliwatts, the gain is approximately

a. 9 db. *b.* 10 db. *c.* 11 db. *d.* 12 db.

57. A certain amplifier has an output resistance (R_o) of 6400 ohms. The input resistance (R_i) of this amplifier is 200 ohms. Using the formula, $\text{db} = 20 \log \dfrac{E_o}{E_i}\sqrt{\dfrac{R_i}{R_o}}$, where $E_i = 0.5$ volts and $E_o = 400$ volts, we find the gain in decibels to be

a. 22 db. *b.* 29 db. *c.* 43 db. *d.* 58 db.

58. A rhombic antenna is supplied with one kilowatt of power, or (P_2), which results in 20 microvolts per meter at the receiving station. A properly oriented half-wave antenna located near the rhombic antenna must be supplied with 16.6 kilowatts, or (P_1), in order to produce the same strength at the receiving station. Using the formula, $\text{db} = 10 \log \dfrac{P_1}{P_2}$, we find the gain of the rhombic antenna to be approximately

a. 1 db. *b.* 12 db. *c.* 22 db. *d.* 112 db.

59. Solution of exponential equations is sometimes necessary in computing problems in electricity. In the equation $4^{x-3} = 42$, we find the unknown or x, equal to

a. 2.7. *b.* 5.7. *c.* 11. *d.* 13.

60. The capacitive reactance of a circuit is given as $X_c = \dfrac{159{,}000}{13.6 \times 470}$. The logarithm of X_c is

a. 1.3958. *b.* 2.3958. *c.* 2.6628. *d.* 3.6628.

LESSON 4

GEOMETRY

CREDIT HOURS - - - - - 2

TEXT ASSIGNMENT - - - TM 11–684, par 128–142, 174–176; Attached Memorandum, par 6

MATERIALS REQUIRED - - None

LESSON OBJECTIVE - - - To familiarize you with lines, angles, and triangles; the measurement of angles; and computation of areas and volumes of geometric figures

SUGGESTIONS - - - - - None

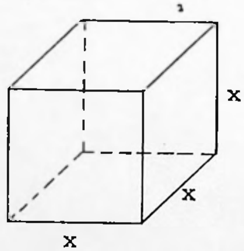
ATTACHED MEMORANDUM

6. SOLID GEOMETRY

a. Uses of Solid Figures. Since we live in a world of three dimensions, many of our practical problems require familiarity with solid figures. In particular, it is often necessary to compute the volume of the solid. The volume is simply the space occupied by the figure. Other situations may require the computation of the surface area of the solid. Some of the more common regular solids are shown below. The formulas for the volume (V) and the surface (S) for each solid are given with the diagrams.

b. Formulas of Common Solids.

(1) Cube (all faces are squares).



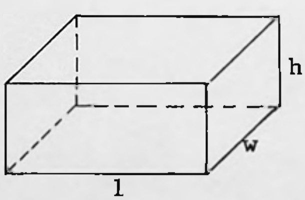
$S = 6x^2$

$V = x^3$

100-3a

Figure 3a. Cube.

(2) Rectangular parallelepiped (all faces are rectangles)



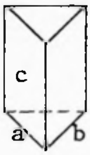
$S = 2(lw + hw + hl)$

$V = lwh$

100-3b

Figure 3b. Rectangular parallelepiped.

(3) Right triangular prism (bases are triangles; lateral faces are rectangles).



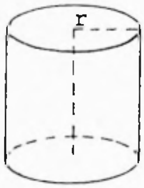
$$S = 2B + h(a + b + c)$$

$$V = Bh$$

where B is the area of the base.

Figure 3c. Right triangular prism.

(4) Right circular cylinder (foot of altitude drawn from center of circular top coincides with center of base).

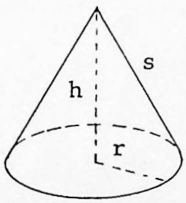


$$S = 2\pi r(r + h)$$

$$V = \pi r^2 h$$

Figure 3d. Right circular cylinder.

(5) Right circular cone (center of circle coincides with foot of altitude from vertex).

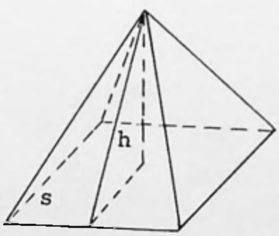


$$S = \pi r(r + s)$$

$$V = \frac{1}{3}\pi r^2 h$$

Figure 3e. Right circular cone.

(6) Regular pyramid (base is a regular polygon with center coinciding with the foot of altitude from vertex).



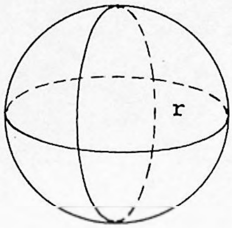
$$S = \frac{1}{2}ps + B$$

$$V = \frac{1}{3}Bh$$

where p is the perimeter of the base and B is the area of the base.

Figure 3f. Regular pyramid.

(7) Sphere.



$$S = 4\pi r^2$$

$$V = \frac{4}{3}\pi r^3$$

Figure 3g. Sphere.

EXERCISES

In each of the following exercises, select the ONE answer that BEST completes the statement or answers the question. Indicate your solution on the answer sheet by filling in with pencil the appropriate lettered block.

All exercises of this lesson are of equal weight. The total weight is 100.

SITUATION

Assume that a V-antenna has been constructed according to the plan shown in figure 4. The distance between poles B and C is found to be 375 feet and the perpendicular distance from pole A to the line joining B and C is 520 feet.

Exercises 61 and 62 are based on the above situation.

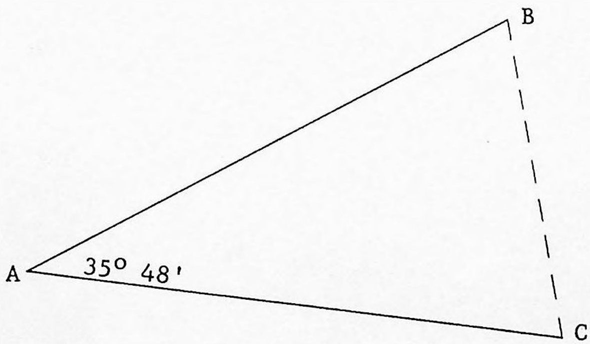


Figure 4. Ground plan of a V-antenna.

61. Since the two legs (AB and AC) of the antenna are equal, the angles at B and C are also equal. The size of each of these angles is

a. 35° 48′. *b.* 36° 03′. *c.* 71° 36′. *d.* 72° 06′

62. The ground area bounded by the lines joining A, B, and C is

a. 4,475 sq ft. *b.* 19,500 sq ft. *c.* 39,000 sq ft. *d.* 97,500 sq ft.

63. Assume that you are to lay a minefield to secure an area that is triangular in shape with equal sides of 175 feet. The angles of the pattern are each 60°. Therefore, the pattern to be used is a/an

a. equilateral triangle.
b. isosceles triangle.
c. right triangle.
d. oblique triangle.

64. A motor messenger is to take a message to three platoon leaders located at points A, B, and C figure 5. You know that the distance "a" is 6 miles, distance "c" is 10 miles, but you must know the distance "b" in order to compute the time needed for the trip. The distance from A to C, or side B is

a. 7 miles.
b. 8 miles.
c. 8.5 miles.
d. 9 miles.

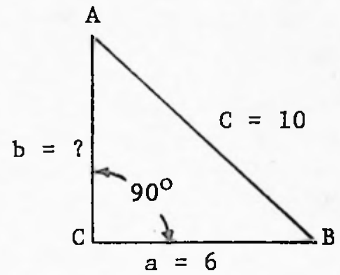


Figure 5. Motor messenger route.

65. A dead-end pole in a field-wire line is to be guyed as shown in figure 6. If 3 feet are allowed at each end of the guy for tying, the length of wire required for the guy is approximately

a. 33 ft.
b. 39 ft.
c. 45 ft.
d. 51 ft.

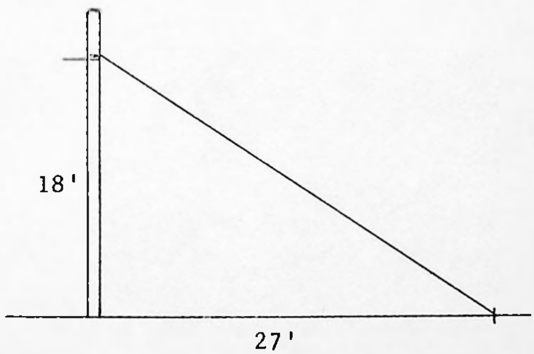


Figure 6. Diagram of pole with guy wire.

66. A division rear command post is to occupy a position which is 1,000 yards on two sides and 500 yards on the other two sides (fig 7). Thus, the position occupied by the division rear command is an area in the form of a

a. parallelepiped.
b. rectangle.
c. square.
d. trapezoid.

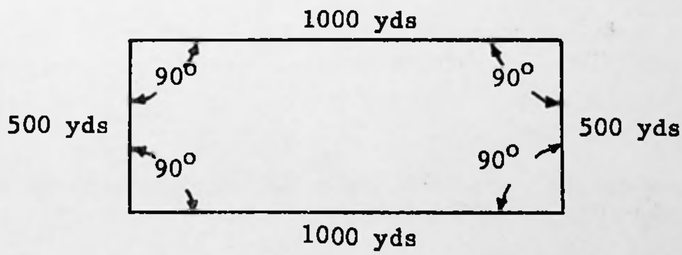


Figure 7. Division rear command post area.

67. Assume that the radiation pattern of an FM voice radio with a vertical antenna is a perfect circle with the transmitter antenna at the center. If the transmitter is effective up to a range of 6½ miles, the area of coverage is approximately

a. 30 sq miles.
b. 130 sq miles.
c. 180 sq miles.
d. 400 sq miles.

68. Assume that you are required to requisition a fence to enclose a circular clearing that has a diameter of 78 feet. If the fencing is issued in 100 ft rolls, the number of rolls you must order is

a. 1. *b.* 2. *c.* 3. *d.* 4.

69. Assume that you are supervising the installation of a rhombic antenna (fig 8) and are required to submit the area of the site to your headquarters. Your crew measures the perpendicular distance from pole D to the opposite side (BC) and finds it to be 295 feet. The area enclosed by the antenna (parallelogram ABCD) is approximately

a. 1.2 acres. *b.* 1.4 acres. *c.* 2.4 acres. *d.* 2.8 acres.

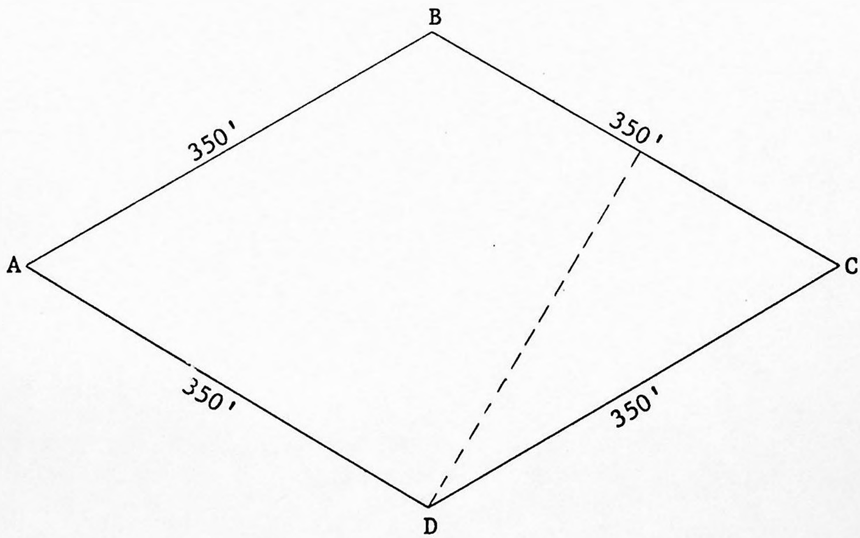


Figure 8. Ground plan of a rhombic antenna.

70. Radar Set AN/MPQ-10 is used to detect objects at distances greater than 450 meters but less than 18,000 meters. The area of the ring of coverage at ground level is approximately

a. 325 sq km.
b. 500 sq km.
c. 810 sq km.
d. 1,000 sq km.

71. Assume that the pattern of coverage of a radar set is a circular sector having a central angle of 0.43 radian. The length of the arc of the sector at a range (radius) of 60 miles is

a. 26 miles.
b. 36 miles.
c. 71 miles.
d. 140 miles.

72. Assume that a generator makes 4,200 revolutions in one minute. The number of radians through which this generator turns in 1 second is

a. 70π radians.
b. 140π radians.
c. 350π radians.
d. 700π radians.

73. When making computations involving angles it is often necessary to convert degrees into radians. The size of a 32° angle in radians is

a. 0.11 radian.
b. 0.19 radian.
c. 0.38 radian.
d. 0.56 radian.

74. After an aircraft at a range of 5,000 meters has been destroyed, the gun is directed toward a second target at a range of 5,000 meters. If the length of the arc (center at the gun) through the two targets is 4,200 meters, the angle through which the gun is moved is

a. 48° 8′.
b. 68° 13′.
c. 84°.
d. 119°.

75. In preparing to move your unit you are required to submit a report on the volume requirements of your equipment. Included in your report will be the total volume of 28 radio sets (rectangular parallelepipeds) each of which measures 14¾ in. × 4¾ in. × 4¼ in. The volume of these 28 sets is approximately

a. 3 cu ft.
b. 5 cu ft.
c. 14 cu ft.
d. 25 cu ft.

76. Assume that you have acquired a cylindrical tank and wish to determine its volume. The diameter of the circular end is 3½ feet and the length is 8½ feet. The volume is approximately

a. 78 cu ft.
b. 82 cu ft.
c. 85 cu ft.
d. 89 cu ft.

77. Assume that you are required to requisition paint to cover the surface of a dome that has the shape of a hemisphere (one-half sphere). If the distance across the floor (diameter of sphere) is 38.0 ft, the exterior surface of the dome is approximately

a. 120 sq ft.
b. 570 sq ft.
c. 1,140 sq ft.
d. 2,270 sq ft.

78. Assume that you are to store 25,000 cubic feet of supplies in pyramidal tents like that shown in figure 9. The floor of the tent measures 16 feet by 16 feet, the wall is 3 feet high, and the height at the center of the tent is 12 feet. If the tents are to be filled completely, the minimum number of tents required is

a. 17.
b. 20.
c. 25.
d. 33.

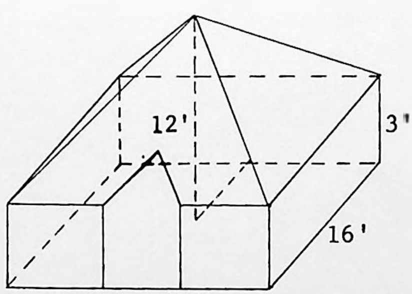


Figure 9. Pyramidal tent.

79. You are responsible for telephone commmunications in an area that has the shape of a trapezoid. The bases of the trapezoid are 14 and 12 kilometers respectively, and the altitude is 10 kilometers. The total area of the trapezoid is

a. 130 sq km.
b. 120 sq km.
c. 110 sq km.
d. 52 sq km.

80. An aerial observation of an Army heliport shows up on the map as a perfect ring. The diameter of the inside circle, or the tower area, is 50 yds. The diameter of the outside circle is 80 yds. Thus, the area of the ring is approximately

a. 1,250 sq yds.
b. 1,980 sq yds.
c. 3,062 sq yds.
d. 4,000 sq yds.

LESSON 5

TRIGONOMETRY—I

CREDIT HOURS - - - - 2

TEXT ASSIGNMENT - - - TM 11–684, par 143–164; review par 133

MATERIALS REQUIRED - - None

LESSON OBJECTIVE - - - To familiarize you with the trigonometric functions, the use of trigonometric tables, and the solution of problems involving the right triangle

SUGGESTIONS - - - - In any exercise where no figure is given, you should sketch the figure to clarify the relationships in your mind.

EXERCISES

In each of the following exercises, select the ONE answer that BEST completes the statement or answers the question. Indicate your solution on the answer sheet by filling in with pencil the appropriate lettered block.

All exercises of this lesson are of equal weight. The total weight is 100.

81. A cable-line pole is to be guyed as shown in figure 10. The height of the pole is 16.0 feet and the guy wire (after being installed) is 19.0 feet. The distance from the base of the pole to the anchor is

a. 9.0 feet.
b. 10.2 feet.
c. 17.4 feet.
d. 24.8 feet.

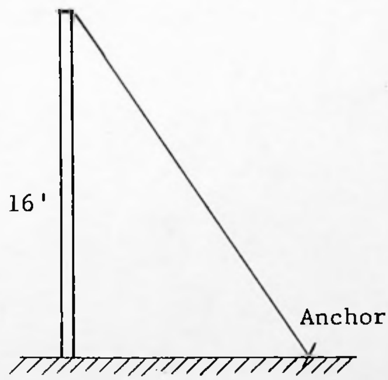


Figure 10. Guyed cable-line pole.

SITUATION

While laying a cable line you have found it necessary to determine the distance across an uncharted body of water (distance BC in fig 11). You accomplish this by laying out the right triangle ABC shown in the figure. Line AC measures 250 yards and the tangent of angle A is $\frac{9}{14}$.

Exercises 82 and 83 are based on the above situation.

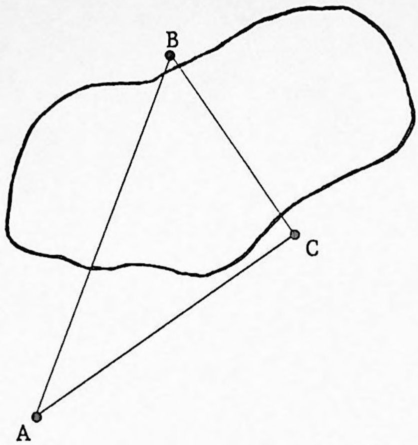


Figure 11. Method of measuring distance across uncharted body of water.

82. The distance across the pond shown in figure 11 is

a. 161 yds.
b. 209 yds.
c. 276 yds.
d. 389 yds.

83. The cosine of angle A shown in figure 11 has a value of

a. 0.64.
b. 0.84.
c. 1.19.
d. 1.56.

84. A map shows that the roads connecting three towns (A, B, and C) form a triangle as shown in figure 12. You are required to send a team from A to C but since the road from A to C is under enemy fire, you instruct the team to travel by way of B. If the direct distance from A to C is 11½ miles, how much farther is the trip by way of B?

a. 4.9 miles
b. 6.6 miles
c. 8.4 miles
d. 19.9 miles

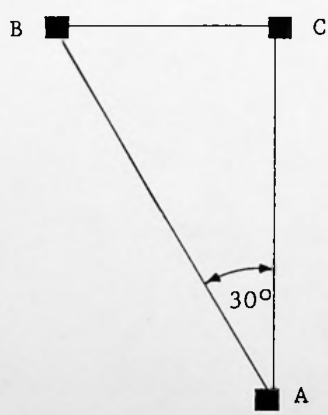


Figure 12. Map of roads connecting towns A, B, and C.

85. The power expended in an ac circuit may be expressed by the Formula $P = EI(\cos b)$ where $\cos b$ is the power factor of the circuit. In some situations it may be convenient to write this formula in its equivalent form which is

a. $P = \dfrac{\sin b}{EI}$.

b. $P = \dfrac{\sec b}{EI}$.

c. $P = \dfrac{EI}{\sin b}$.

d. $P = \dfrac{EI}{\sec b}$.

86. The instantaneous value of electromotive force (e) in an ac circuit is given by the formula $e = E_m \sin a$, where E_m is the maximum value of electromotive force and *a* is the phase angle. If $E_m = 250$ volts, the instantaneous voltage when the phase angle is 45 degrees is

a. 125 volts.
b. 144 volts.
c. 177 volts.
d. 250 volts.

87. A radar set may be used to determine the height of an aircraft as shown in figure 13. If the slant range is 3,100 meters and the angle of elevation is 60 degrees, the height of the target is

a. 895 meters.
b. 1,550 meters.
c. 2,685 meters.
d. 3,580 meters.

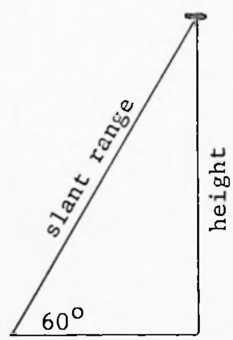


Figure 13. Detection of aircraft with radar.

88. The trigonometric functions are interrelated in many ways. In one such relationship it is found that the sine of a given angle A is equal to

a. $\dfrac{1}{\cos A}$.

b. $\dfrac{1}{\tan A}$.

c. $\cos (90° - A)$.

d. $\tan (90° - A)$.

89. The power factor of an ac circuit is equal to $\cos a$ where *a* is the angle by which the current leads or lags the impressed voltage. If the current lags the voltage by 67°, the power factor is

a. 0.37.
b. 0.39.
c. 0.92.
d. 0.93.

90. The power factor in an ac circuit is equal to the cosine of the angle between the phases of the current and voltage. If the power factor is 83.4%, the phase angle is

a. 4° 47′.
b. 33° 29′.
c. 56° 31′.
d. 85° 13′.

91. Assume that a pilot starts from S and sets his course at a bearing of 41° from true north and that the coast line follows a north-south direction as shown in figure 14. After flying a distance of 450 miles, the aircraft will be offshore by a distance of

a. 295 miles. *b.* 301 miles. *c.* 334 miles. *d.* 340 miles.

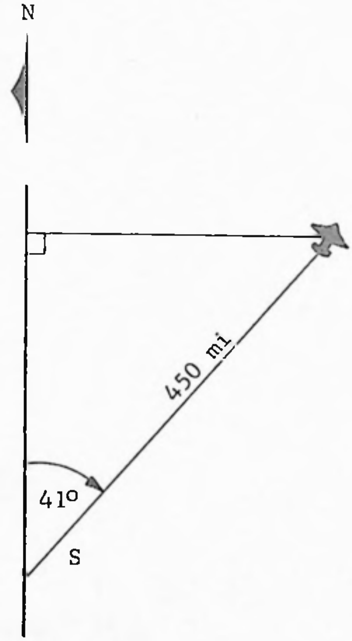


Figure 14. Diagram of aircraft flight pattern.

92. The angle at which a gun must be aimed to place a projectile at its highest point in flight in a given length of time may be found from the formula $\sin a = \frac{tg}{v}$. If $v = 1500$ and $g = 32$, the angle (a) at which the gun must be directed to place the projectile at maximum height when $t = 40$ is

a. 21° 24′. *b.* 31° 25′. *c.* 58° 35′. *d.* 68° 36′.

93. Assume that you are required to determine the distance across a canyon (fig 15). From altimeter readings you determine that the depth at point B is 3,250 feet. If the angle of depression at point 0 is 17° 23′, the distance across the canyon is approximately

a. 10,200 feet. *b.* 10,400 feet. *c.* 10,720 feet. *d.* 10,830 feet.

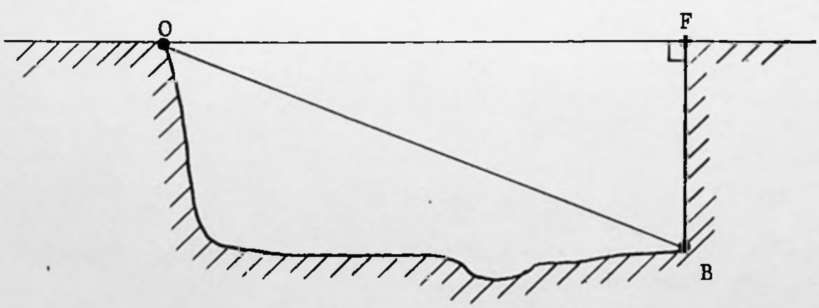


Figure 15. Method of measuring distance across a canyon.

94. A weather balloon being tracked by a ground radar set is found to have an altitude of 4,900 feet when the radar beam makes an angle of 54° 31′ 15″ with the ground. If the balloon was released at the antenna site, the ground distance travelled by the balloon is approximately

a. 2,840 feet.
b. 3,490 feet.
c. 3,990 feet.
d. 6,860 feet.

95. In surveying a parcel of land you find that its slope is a right triangle whose vertices you designate as B, C, and D. Line CD runs from north to south and CB runs from east to west. If CB is 236.4 feet long and DB is 837.2 feet long, the direction of DB makes an angle with north of

a. 15° 46′.
b. 16° 24′.
c. 19° 30′.
d. 20° 44′.

96. Assume that the height of a certain mountain peak is 6,426 feet. From the summit of a hill 2,048 feet high the angle of elevation of the peak is 23° 18′. The distance between the two summits is

a. 4,770 feet.
b. 4,780 feet.
c. 10,670 feet.
d. 11,070 feet.

97. Assume that a radio relay station (B of fig 16) located at an elevation of 130 meters is in radio contact with a station (A of fig 16) at sea level. If the angle of depression from B to A is 7° 10′, the line-of-sight distance (AB) is

a. 970 meters. 700
b. 1000 meters. 960
c. 1030 meters. 1040
d. 1100 meters. 1310

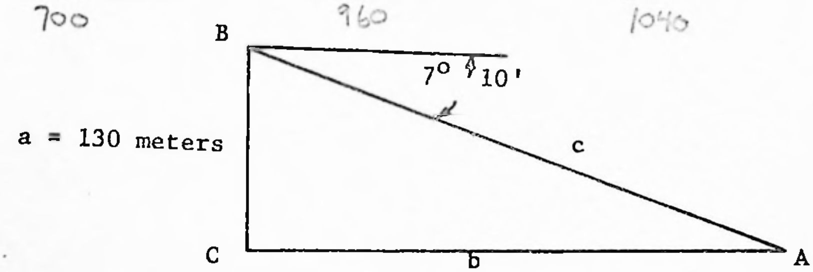


Figure 16. Determining line-of-sight distance between radio relay stations.

98. Find the unknown sides a and b and also the value of angle B in right triangle ABC (fig 17) if angle A is 25° 30′ and hypotenuse c is 9.0 inches.

a. a = 3.9 in; b = 8.1 in; angle B = 64° 30′
b. a = 3.9 in; b = 8.1 in; angle B = 44° 30′
c. a = 4.4 in; b = 7.8 in; angle B = 64° 30′
d. a = 4.4 in; b = 7.8 in; angle B = 44° 30′

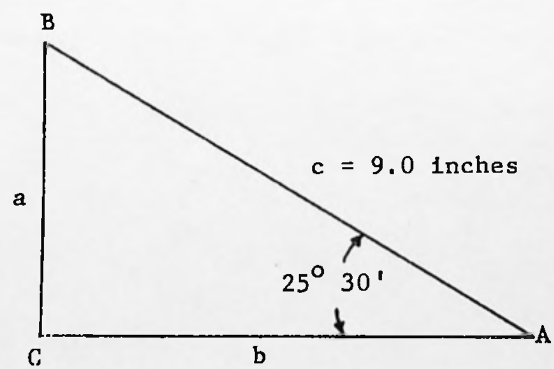


Figure 17. Right triangle.

99. In order to stabilize a recently installed line pole you must prepare and construct a cable support as shown in figure 18. You know the acute angle is 35° 22′ and the adjacent side is 18.0 feet. The length of the cable c (from A to B) must be

a. 10.4 ft.

b. 14.7 ft.

c. 22.1 ft.

d. 30.2 ft.

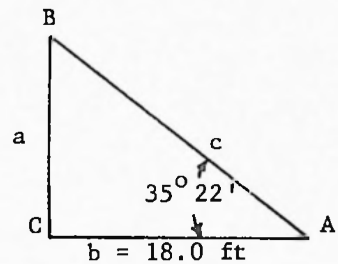


Figure 18. Cable supported line pole.

100. A radio antenna 500 feet high is viewed from a point 200 feet away from the base of the tower (fig 19). The angle of elevation to the top of the tower is

a. 21° 50′.

b. 34° 6′.

c. 43° 40′.

d. 68° 12′.

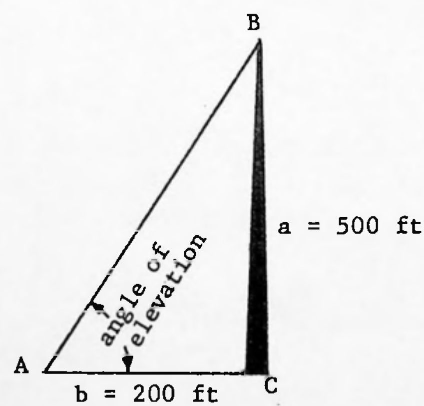


Figure 19. Computing the angle of elevation of antenna mast.

LESSON 6

TRIGONOMETRY—II

CREDIT HOURS - - - - 2

TEXT ASSIGNMENT - - - TM 11–684, par 165–173; Attached Memorandum, par 7

MATERIALS REQUIRED - - None

LESSON OBJECTIVE - - - To show you some of the methods of solving the oblique triangle, including the law of sines, law of cosines, law of tangents, and formulas for finding areas of triangles

SUGGESTIONS - - - - None

ATTACHED MEMORANDUM

7. FUNCTIONS OF ANGLES GREATER THAN 90°

Occasionally it is necessary to find the trigonometric function of an angle that is greater than 90 degrees. Table I shows the equivalent forms of trigonometric functions of various angles between 90° and 360°. For any angle greater than 360°, simply subtract the greatest integral multiple of 360° from the angle and apply the rules to the remainder. For example:

a. Find sin 750°.

$$750^\circ - 2(360^\circ) = 30^\circ$$
$$\sin 30^\circ = 0.5000$$

b. Find cos 1,680°.

$$1{,}680^\circ - 4(360^\circ) = 240^\circ$$
$$\cos 240^\circ = -\cos 60^\circ = -0.5000$$

TABLE I

EQUIVALENT TRIGONOMETRIC FUNCTIONS FOR ANGLES GREATER THAN 90 DEGREES

$\sin(90^\circ + x) = \cos x$ $\cos(90^\circ + x) = -\sin x$	$\tan(90^\circ + x) = -\cot x$ $\cot(90^\circ + x) = -\tan x$
$\sin(180^\circ + x) = -\sin x$ $\cos(180^\circ + x) = -\cos x$	$\tan(180^\circ + x) = \tan x$ $\cot(180^\circ + x) = \cot x$
$\sin(270^\circ + x) = -\cos x$ $\cos(270^\circ + x) = \sin x$	$\tan(270^\circ + x) = -\cot x$ $\cot(270^\circ + x) = -\tan x$

EXERCISES

In each of the following exercises, select the ONE answer that BEST completes the statement or answers the question. Indicate your solution on the answer sheet by filling in with pencil the appropriate lettered block.

All exercises of this lesson are of equal weight. The total weight is 100.

101. In mapping large areas, surveyors often divide the area into a series of triangles as shown in figure 20. This method is called *triangulation*. If one of the triangles (ABC) in a given area has been measured and BC = 231.3 rods, angle CAB = 24° 31′, and angle ABC = 76° 14′, distance AC is

a. 497.2 rods.
b. 531.2 rods.
c. 541.4 rods.
d. 547.4 rods.

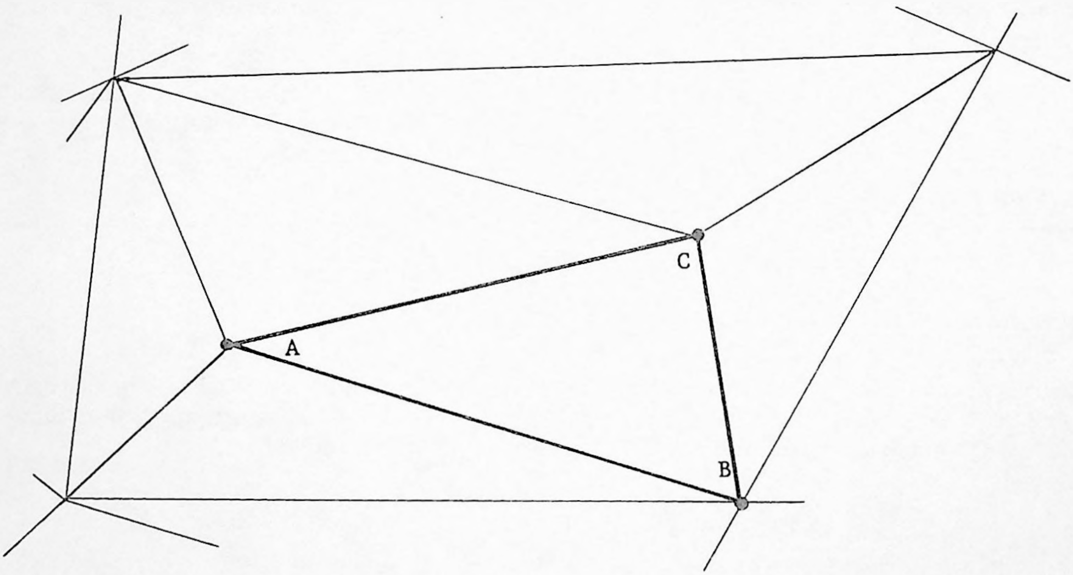


Figure 20. Method of triangulation.

102. One of the poles supporting a cable line is secured by two wires attached to the ground as shown in figure 21. The longest wire is 57.46 feet long and makes an angle of 54° 42′ with the ground. If the shorter wire is 50 feet long, the angle it makes with the ground is

a. 64° 18′.
b. 65° 45′.
c. 67° 22′.
d. 69° 43′.

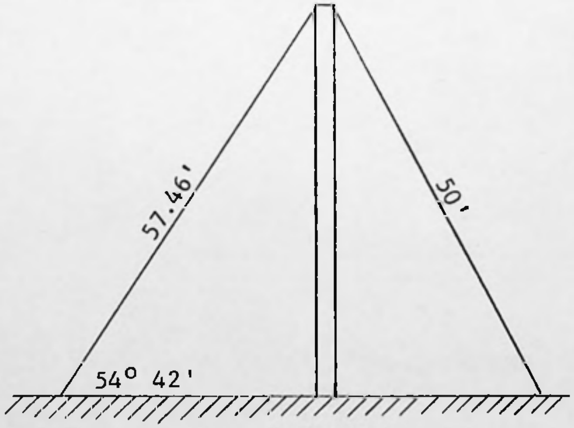


Figure 21. Single pole secured by guy wires.

103. To determine the distance of an enemy position (A) from the observation post (B), 54-meter line (BC) was drawn and angles ABC and BCA measured as 76° and 82° (fig 22). The distance from the observation post (B) to the enemy position is approximately

a. 30 meters.
b. 56 meters.
c. 99 meters.
d. 143 meters.

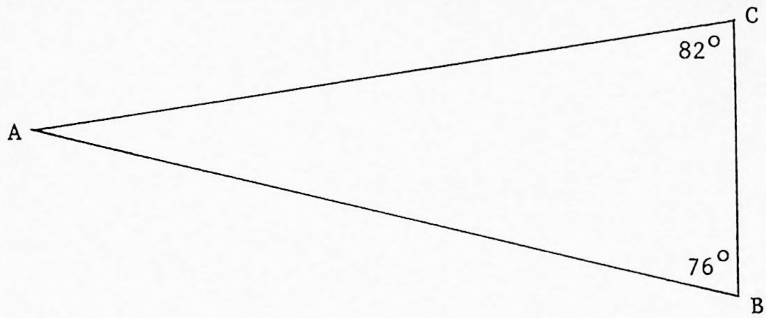


Figure 22. Method of locating enemy position.

104. The distance from gun G to an enemy machine gun nest M can be determined by selecting a reference point P and using the method of triangulation. If the distance GP = 185.6 meters, angle MGP = 59° 18′ and angle GPM = 74° 52′, the distance between G and M is

a. 153 meters.
b. 165 meters.
c. 209 meters.
d. 250 meters.

105. An engineer unit has been ordered to construct a road through a dense woods from point R to point S as shown in figure 23. These points are visible from point T, so measurements are taken as follows: RT = 200 meters, ST = 300 meters, and angle RTS = 30°. The length of the road will be

a. 90 meters.
b. 104 meters.
c. 161 meters.
d. 260 meters.

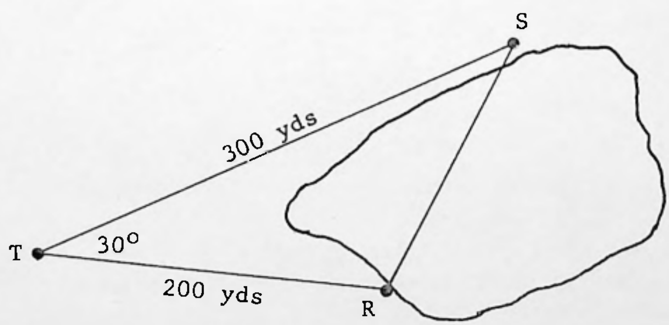


Figure 23. Method of determining distance through dense woods.

106. The sides of a triangular field have been measured at 30.0 meters, 50.0 meters, and 60.0 meters. The size of the angle opposite the shortest side is

a. 29° 56′.
b. 31° 11′.
c. 54° 46′.
d. 60° 04′.

107. Assume that a viewer is seated in front of a television set that has a diagonal screen width of 21 inches (fig 24). If the distance from the eye to each end of the diagonal is 16.0 feet, the viewing angle is

a. 6° 16′. *b.* 8° 47′. *c.* 14° 22′. *d.* 19° 16′.

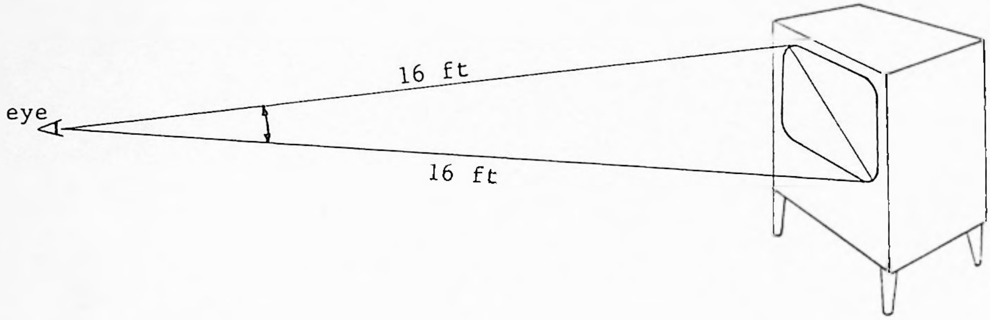


Figure 24. Method of determining a television viewing angle.

108. The distance across a pond is to be determined. Two points (A and B on opposite sides of the pond) are chosen and a nearby flagpole (P) selected as a reference. Distance AP = 200 ft, distance BP = 160 ft, angle B = 51° 21′, and angle A = 38° 39′. The distance across the pond is

a. 41 ft.
b. 120 ft.
c. 256 ft.
d. 360 ft.

Suggestion:
Use Law of Cosines

109. The forces affecting the speed and direction of an airplane may be represented by vectors as shown in figure 25. If the air speed is 230 mph, the wind velocity 20 mph, and angle WGA = 150°, the angle of drift is approximately

a. 2°. *b.* 4°. *c.* 6°. *d.* 8°.

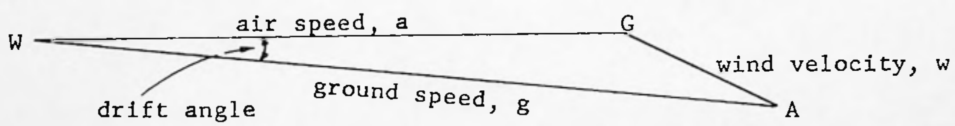


Figure 25. Vector diagram of velocities affecting airplane flight.

110. Assume that a relay station is required at point C to establish radio communication between points A and B. (See figure 26.) If distance AC = 17.4km, distance BC = 21.7km and angle ACB = 140°, the logarithm of tan ½(A − B) is

a. 2.7475. *b.* 1.5611. *c.* 8.6024 − 10. *d.* 8.4399 − 10.

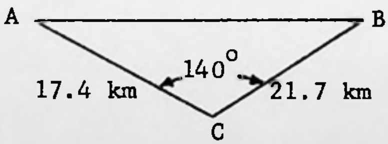


Figure 26. Radio relay systems.

111. Two electrical forces of 62.4 volts and 48.0 volts are applied in the directions shown in figure 27. If the angle between the two applied forces is 120°, the angles between the applied forces and the resultant force are

a. 28° 36′ and 31° 24′.
b. 34° 18′ and 25° 42′.
c. 37° 12′ and 22° 48′.
d. 37° 34′ and 22° 26′.

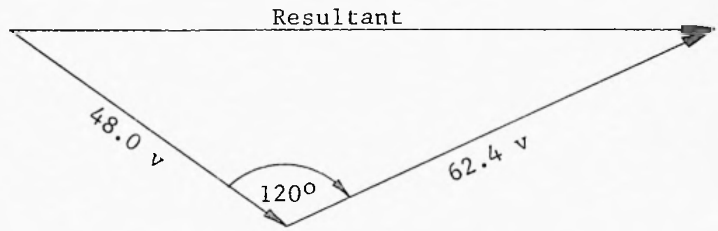


Figure 27. Vector diagram of two voltages and resultant.

112. A tunnel is to be dug through a hill and the distance from point P at one end to point Q at the other end must be determined (fig 28). A third point R is chosen so that P and Q are both visible from it and measurements are taken. One of the steps involved in computing the distance PQ, using the Law of Tangents is

a. $(P + Q) = 60° 46'$.
b. $(P - Q) = 14° 15'$.
c. $\tan \frac{1}{2} (P - Q) = \dfrac{1.78685 \times 227.2}{1596.4}$.
d. $r = \dfrac{911.8 \times \sin 75° 1'}{\sin 58° 28'}$.

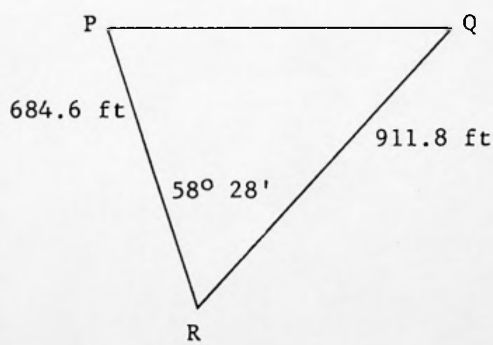


Figure 28. Method of determining length of proposed tunnel.

113. A force of 812.6 lbs applied in direction AB (fig 29) is acted upon by a force of 900.0 lbs in direction BC. The resultant force of 1257.0 lbs has a direction AC. The direction of the resultant force (AC) differs from that of the original force (AB) by an angle of

a. 41° 9′.
b. 42° 47′.
c. 45° 34′.
d. 47° 18′.

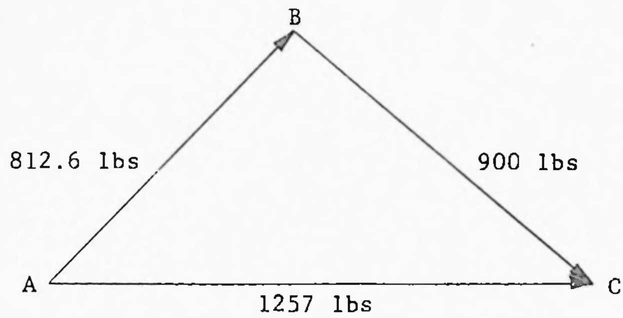


Figure 29. Diagram of force vectors.

114. Assume that you are required to determine the area of a triangular field that may be used as a radio site. After measuring the sides and finding them to be 640.3 ft, 800.6 ft, and 564.9 ft, you find the area to be

a. 0.44 acre.
b. 1.58 acres.
c. 3.15 acres.
d. 4.12 acres.

115. In reconnoitering an area, you find it necessary to determine the area of a triangular plot of ground that is covered with a dense woods. You are able to measure two of the sides at 200 yds and 300 yds. If the included angle is found to be 34° 19′, the area is

a. 2.6 acres.
b. 3.5 acres.
c. 5.1 acres.
d. 7.0 acres.

116. Assume that two ships, separated by 50 miles, are located at points A and B (fig 30). They have learned that a flyer has been downed in the triangular area ABC. If angle CAB = 67° and angle ABC = 58°, the area of search is approximately

a. 950 sq miles.
b. 1,100 sq miles.
c. 1,200 sq miles.
d. 1,350 sq miles.

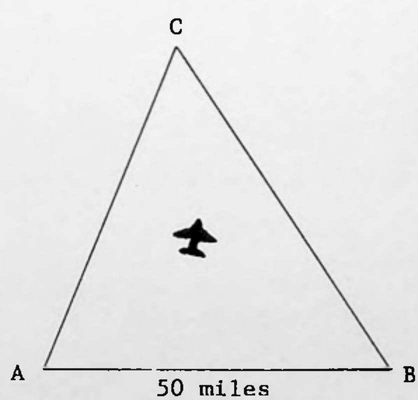


Figure 30. Method of determining triangular area of search.

117. Assume that a wire team has been ordered to install telephone connections between command areas A, B, and C shown in figure 31. Use the Law of Cosines to find the distance from A to C.

a. 9 km
b. 11 km
c. 13 km
d. 14 km

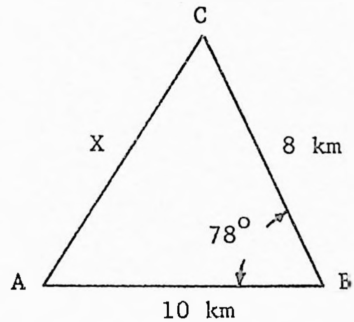


Figure 31. Command area.

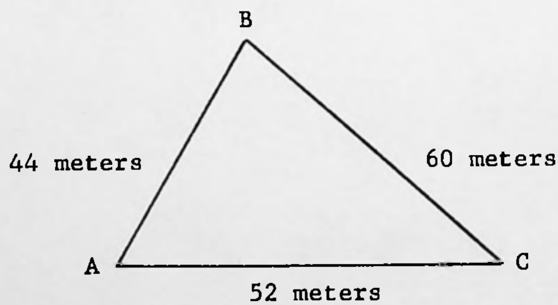
118. A reconnaissance party has been given a triangular area to reconnoiter prior to occupation. The dimensions of the area are 4 km by 8 km by 10 km. The total area to be covered by the reconnaissance party is approximately

a. 13 sq km.
b. 14 sq km.
c. 15 sq km.
d. 16 sq km.

119. Assume that a helicopter has been assigned a prescribed area for surveillance. The area, when drawn on a map, forms an oblique triangle with sides of 8 and 10 km respectively, and an included angle of 30° 30′. The area covered by the helicopter is

a. 19 sq km.
b. 21 sq km.
c. 22 sq km.
d. 23 sq km.

120. As platoon leader you are given an area to set up your platoon CP tent. You are to occupy a triangular area with sides of 60 meters, 52 meters, and 44 meters. However, you must know the angles of the area (see figure 32) in order to situate your CP in the area. Thus, the angles A, B, and C of the triangle are equal to



a. A = 38° 26′
B = 57° 34′
C = 74°.

b. A = 38° 26′
B = 96°
C = 45° 34′.

c. A = 76° 52′
B = 56° 47′
C = 46° 21′.

d. A = 76° 52′
B = 57° 34′
C = 45° 34′.

Figure 32. Platoon area (not drawn to scale).

LESSON 7

FUNCTIONS AND GRAPHS

CREDIT HOURS - - - - 2

TEXT ASSIGNMENT - - - TM 11–684, par 95–100, 144; Attached Memorandum, par 8–12

MATERIALS REQUIRED - - None

LESSON OBJECTIVE - - - To familiarize you with the types of mathematical relationships that are useful in the field of communication—algebraic, exponential, logarithmic, and trigonometric functions

SUGGESTIONS - - - - Review all previous lessons

ATTACHED MEMORANDUM

8. FUNCTIONAL RELATIONSHIPS

a. General. Change is inevitable. We grow, life changes, and the world around us changes. We are constantly aware of the changes taking place about us. Some of the changes—like the position of the sun and the phases of the moon—are predictable; other changes—like earthquakes and tornados—are unpredictable. We often try to account for a change in a situation, to explain it, or to relate it to some other occurrence. For example, we attempt to explain a change in the cost of living by comparing it with other changes such as those in profits or wages, efficiency of production, and the supply and demand for goods. In this section we shall study the basic ideas of relations among changing quantities.

b. Constants. Mathematics deals with two kinds of quantities or magnitudes—constants and variables. A *constant* is a quantity that retains the same value throughout a given problem or discussion. Absolute constants, such as 5, -7, π, etc., have the same values in all problems. Arbitrary constants, such as the radius of a circle, may have different values in different problems.

c. Variables and Functions. It often happens that two variables in a situation are so related that for each value of the first a corresponding value of the second is determined. For example, if a vehicle moves directly away from a point P at the constant rate of 40 miles per hour, its distance from P varies all the way from 0 to 40 miles during the first hour. During this period the distance from P has every intermediate value, and consequently an unlimited number of values in all. The relation between the distance and the time may be expressed algebraically by an equation or formula, such as $y = 40x$, in which y represents the distance traveled in miles, and x the number of hours. A value of y can be found for any value of x as shown in table II. In this equation, both x and y are *variables,*

TABLE II

VALUES IN $y = 40x$

x	0	1	2	3	5
y	0	40	80	120	200

and y is said to be a function of x. That is, the value of y depends upon the value of x. Here x is regarded as the independent variable and y as the dependent variable. The idea of function is found often in everyday experience, and is very important in mathematics.

d. Method of Representing Functional Relationships. We have already seen that the relation between variables can be shown by a mathematical formula or equation, and by means of a table. When a sufficiently extensive table of corresponding values is given, a *graph* can be drawn. Usually the values have been obtained by computation, actual counting, or careful measurement. Figure 33 shows a simple graph that has been made from a table of values. Each pair of values is used to locate a point on the graph. The points are then connected by a line that is as smooth as possible. It is ap-

Current (amps)	2.5	5.0	7.5	10.0	12.5	15.0	17.5
Potential (volts)	18.8	37.5	56.3	75.0	93.8	112.5	131.3

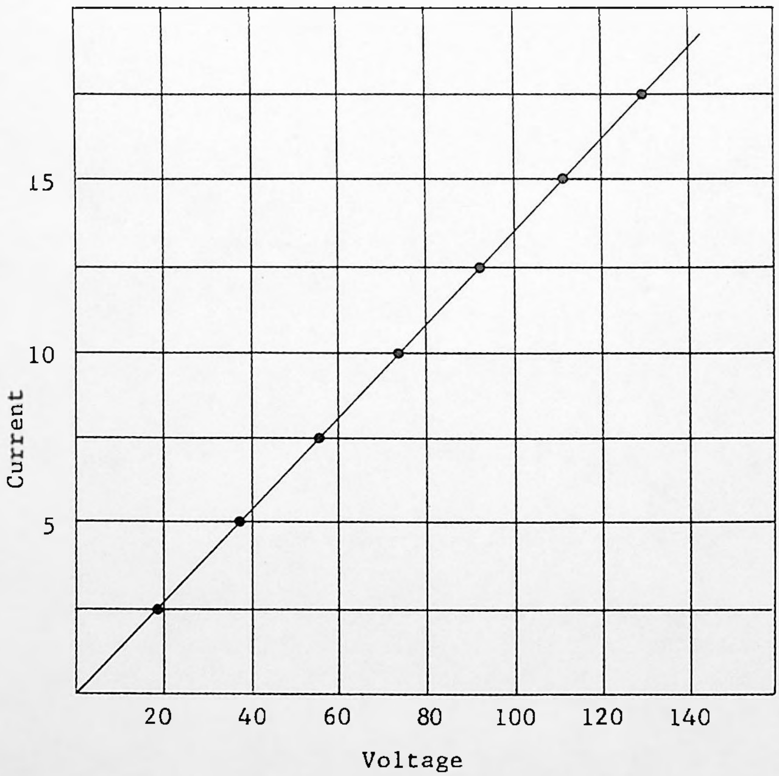


Figure 33. Table and graph showing relationship between voltage and current in a circuit having a resistance of 7.5 ohms.

parent that graphic methods exhibit the actual variation of a function more clearly than do tables of values. For this reason, in scientific research the data are often put on graphs to be studied for general trends as well as for peculiarities at certain points. In this way underlying principles frequently are discovered.

9. GRAPHING METHODS

a. Directed Lines. In laying off distances from a point on a straight line, it is often useful to have some way of determining which direction is to be taken. This may be done by placing an arrowhead on the line to indicate direction: ⟶ or ⟵ . Direction may also be indicated by prefixing a label or sign to each number used; (+) if the distance is taken in one direction, and (−) if in the opposite direction. Thus, if distances to the right are taken as positive, distances to the left are negative; if up is taken as positive, down is taken as negative; if the direction A to C is taken as positive, C to A is taken as negative. In geometry, whether we speak of a line-segment as AB or BA, the meaning remains unchanged. In a directed line, however, if AB represents a positive number, BA is negative.

b. Axes and Quadrants. Two lines perpendicular to each other and lying in the same plane may be taken as lines of reference or *axes.* These lines are usually designated by XX′ and YY′, and are called the x-axis and y-axis respectively (fig 34). The point of intersection, O, is called the *origin.*

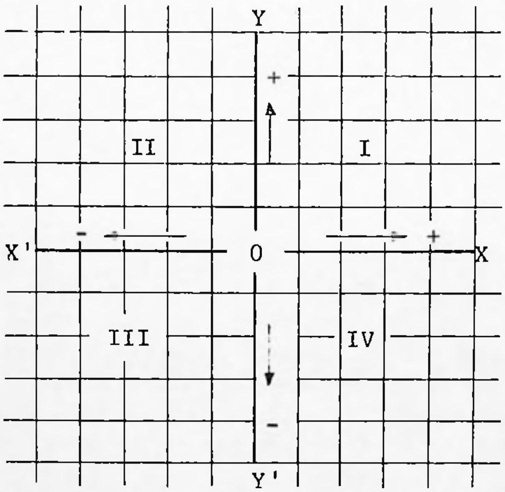


Figure 34. Axes and quadrants of a graph.

It serves as a convenient starting point from which to measure distances. The two axes divide the plane into four parts called *quadrants.* The part of the plane which lies above the x-axis and to the right of the y-axis is called the first quadrant, and is designated by I. The other quadrants are numbered II, III, and IV as shown in figure 34.

c. Rectangular Coordinates of a Point. In the graph shown in figure 35 we will choose a point P and draw a line parallel to the y-axis from P to N on the x-axis. Then ON is the x-distance or *abscissa* of the point P, and NP is the y-distance, or *ordinate.* Together the abscissa and ordinate are called the coordinates. For brevity, they are written within parentheses and separated by a comma, the abscissa always being written first, as (x, y). Paper ruled in squares, called coordinate paper, is

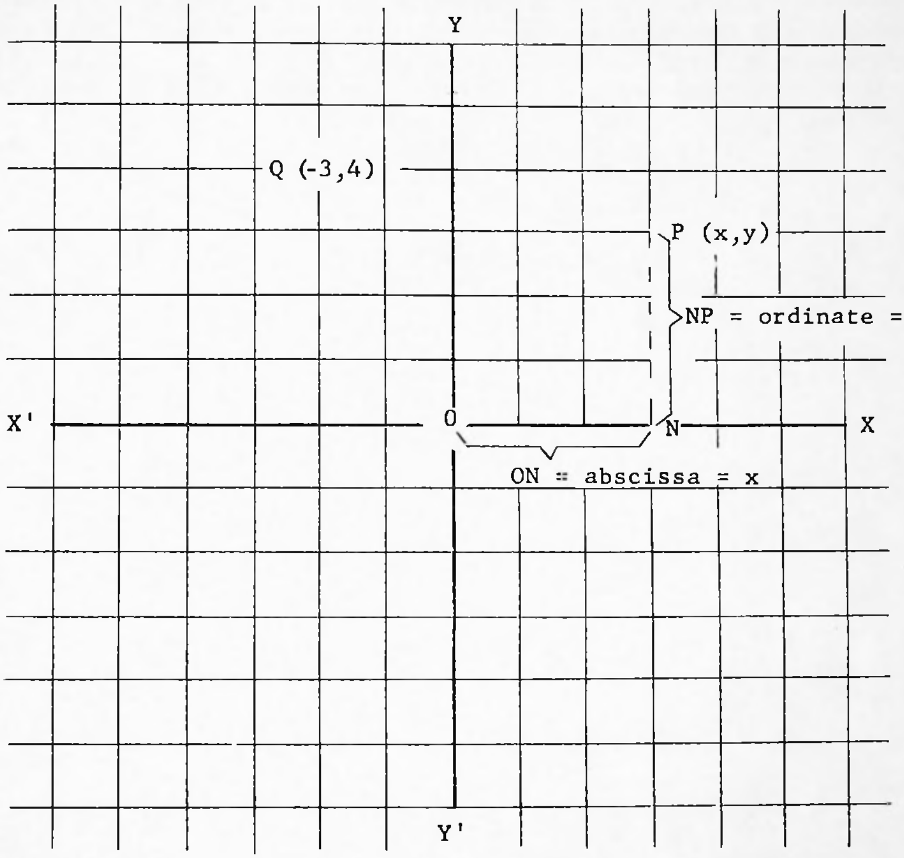


Figure 35. Locating points on a graph with rectangular coordinates.

convenient when we are working with rectangular coordinates. Locating points on a graph by means of their coordinates (sometimes called ordered pairs) is called plotting the points. Thus, the ordered pair (—3, 4) locates point Q on the graph, the point being three units to the left of the y-axis and four units above the x-axis.

d. Signs of the Coordinates. Abscissas are considered positive if measured to the right of the y-axis and negative if measured to the left. Ordinates are considered positive if measured upward from the x-axis and negative if measured downward from the x-axis. The signs of the coordinates in the different quadrants are shown in table III.

TABLE III

SIGNS OF COORDINATES BY QUADRANTS

Quadrants	I	II	III	IV
Abscissa	+	—	—	+
Ordinate	+	+	—	—

e. Graphic Representation of Functions. To construct a graph of a function, values of the variable (x) are taken as abscissas of the points, and the corresponding values of the function (y) are taken as the ordinates. A smooth curve which contains these points in order is called the graph of the function. For example, let us construct the graph of $y = 3x - 4$. First, we will select appropriate values of x and find the corresponding values of y. Next, we place the values in a table (table IV).

TABLE IV

VALUES OF x and y FOR $y = 3x - 4$

If	$x =$	-2	-1	0	1	2	3	4
Then	$y =$	-10	-7	-4	-1	2	5	8

Now we plot the points on the graph (fig 36). You will note that the graph of this function is a straight line. Equations with straight-line graphs are called *linear equations.*

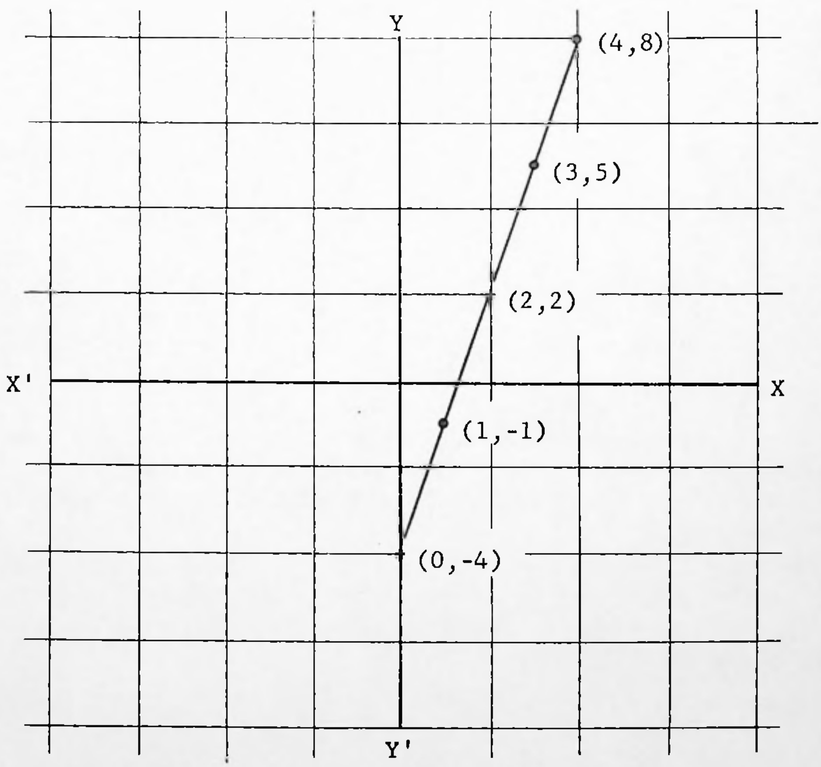


Figure 36. Graph of $y = 3x - 4$.

10. VARIATION OF FUNCTIONS

a. General. In scientific and technical work it is common to find that a change in one variable produces a change in another. Thus the growth of a plant depends upon the soil, light, temperature, and humidity. The force between two electric charges depends on the size of the charges and the distance between them. The voltage across a resistor depends on the current flowing through the resistor.

b. Direct Variation. The simplest relationship that exists between two variables is expressed by the equation $y = kx$. In this relation each variable is said to *vary directly as* the other. That is, the doubling of one doubles the other and so on. Relations of this kind occur very frequently in everyday life. For example, if k is the speed of vehicle, x is the time of travel, and y is the distance traveled, then $y = kx$. If k is the current in a circuit, x is the resistance, and y is the voltage, then $y = kx$ (better known as Ohm's law, $E = IR$).

(1) The letter k is known as the constant of variation. If $y = kx$, then y varies directly as x. The graph of this equation is a straight line through the origin, and the direction of the line depends on the value of k. Figure 37 shows the graph of $y = kx$ where k is given different values ($\frac{1}{4}$, 1, 4, -2). In practice, the constant k is determined by some condition in the problem.

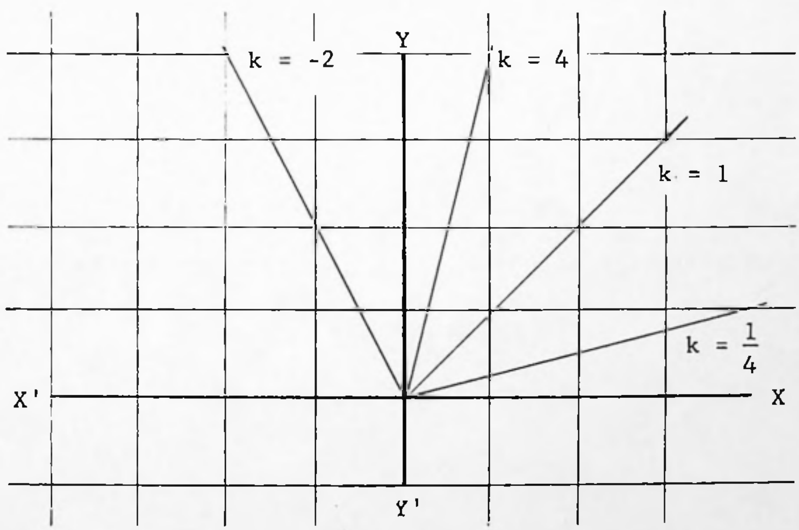


Figure 37. Graph of $y = kx$.

(2) If $y = kx + a$, the graph will be a straight line that does not go through the origin. Figure 38 shows the graph of $y = kx + a$ where $a = 1$, and $k = 2$. You should draw other graphs for this equation by substituting different values for a and k.

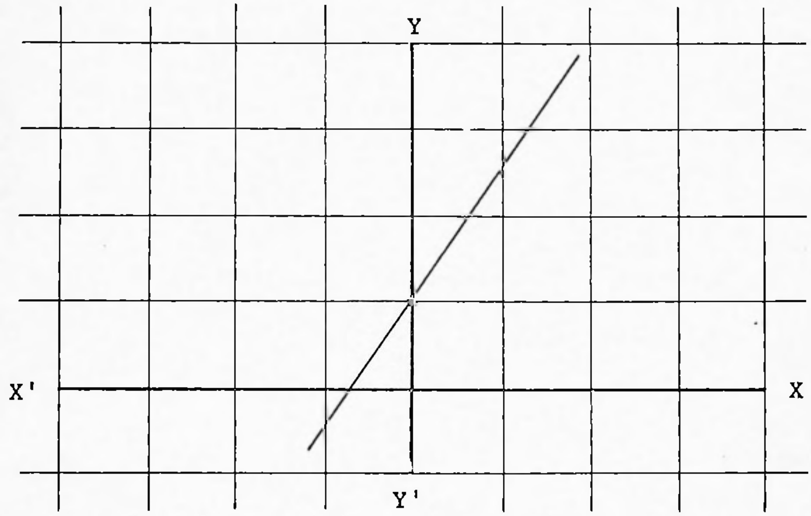


Figure 38. Graph of $y = kx + a$.

(3) Another type of direct variation is given by the equation $y = kx^2$. In this relationship, if x is doubled, y is increased four times. Figure 39 shows the graph of this function for $k = 1/4$ where $y = \frac{1}{4}x^2$, or $x^2 = 4y$. The graph of this function is called a *parabola.*

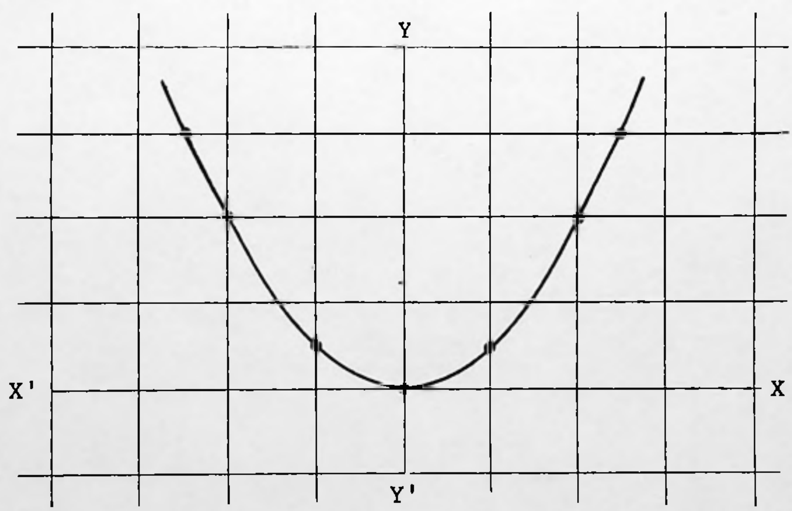


Figure 39. Graph of $x^2 = 4y$.

c. Inverse Variation. When the product of two variables is constant, as $xy = k$, each variable is said to *vary inversely as* the other. Since this equation may also be written as $y = \frac{k}{x}$, or $x = \frac{k}{y}$, each variable may be said to vary as the reciprocal of the other. The graph of this relationship is called a *rectangular hyperbola.* As in the case of the straight line, one ordered pair of values is sufficient to determine k and fix the graph.

(1) Boyle's Law states that the volume (V) of a given quantity of gas at a constant temperature varies inversely as the pressure (P). That is, $PV = k$. Figure 40 shows the graph of $PV = k$, where $k = 6$.

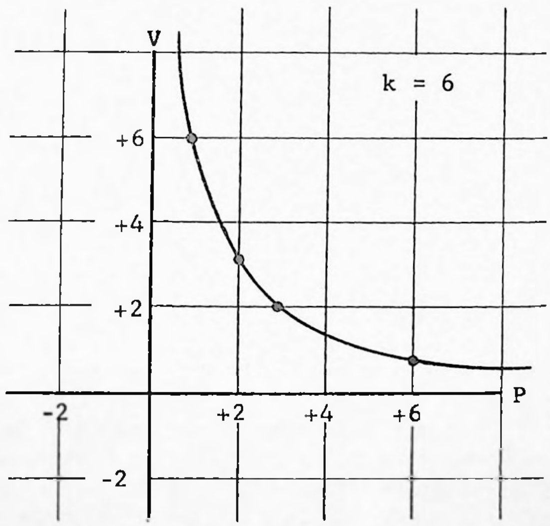


Figure 40. Graph of $PV = k$.

(2) The intensity (I) of light varies inversely as the square of the distance (d) of the source. That is, $Id^2 = k$, or $I = \frac{k}{d^2}$. Figure 41 (page 42) shows the graph of $Id^2 = k$, where $k = 16$.

d. Joint Variation. One quantity is said to *vary jointly* as two or more others when it varies directly as their product. For example, the interest (I) earned on an investment varies jointly with the size (P) of the investment, the rate (r) of interest and the length (t) of time. Thus, $I = Prt$. The interest may be increased by increasing the amount of the investment, the rate of interest, or the time. Likewise, the interest can be decreased by decreasing P, r, or t. If $Z = kxy$, Z is said to vary jointly as x and y. If $Z = \frac{kx}{y}$, Z is said to vary directly as x and inversely as y. Finally, any number of variables may be found in a joint relationship.

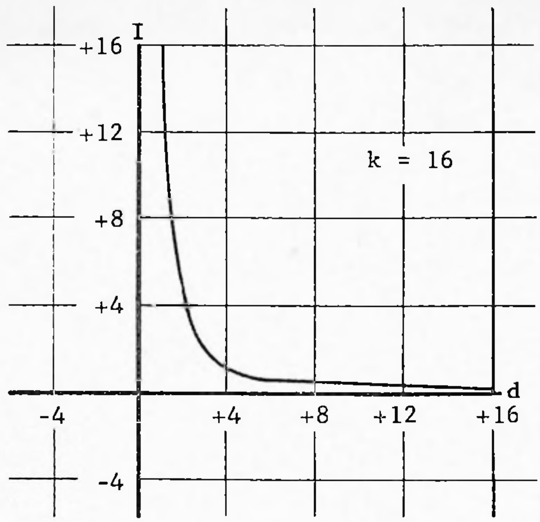


Figure 41. Graph of $Id^2 = k$.

11. OTHER TYPES OF FUNCTIONS

a. Exponential Functions.

(1) In certain situations it is observed that when one variable changes, the other variable changes at a much different rate. For example, we may consider certain types of bacteria that increase in number by the process of subdivision. Table V shows the number of bacteria (Q) which will result from the original bacterium at the end of any number of hours (t). At the end of t hours there are clearly 2^t bacteria. Thus, the relation between Q and t is expressed by the equation $Q = 2^t$. The equation representing this kind of relation is called an *exponential equation.* It should be noted that the independent variable t appears as the exponent of a constant base. If we had started with five bacteria in the above example there would have been just five times as many at the end of t hours, or in the equation form, $Q = 5(2^t)$. If we start with three bacteria, the equation is $Q = 3(2^t)$, and if we start with Q_o bacteria, the equation is $Q_o(2^t)$.

TABLE V

RELATION BETWEEN ELAPSED TIME AND NUMBER OF BACTERIA

t (hours)	0	1	2	3	4	5	6	7	–	–	–	–
Q (bacteria)	1	2	4	8	16	32	64	128	–	–	–	–

(2) The increase of bacteria is only one example of a fairly general law of growth in nature, for it has been observed that many quantities in nature increase in this manner during some period of their existence. The form of exponential equation that is most frequently found in electrical work is based on the *standard exponential equation,* $y = e^x$. The letter e is a special constant having a value of 2.71828. Other exponential equations closely related to $y = e^x$ are the equations $y = ke^x$ and $y = ke^{ax}$.

(3) The graphs of exponential functions can be made in the usual manner of listing the corresponding values and plotting each point. Figure 42 shows several typical *curves.* The

curve $y = 2^x$ was drawn using the data in table VI and illustrates the relationship $Q = 2^t$. The remaining graphs are of the form $y = ke^{ax}$ where a and k have arbitrarily assigned values. Thus, if $a = 2$ and $K = 1$, the graph will be as shown for the equation $y = e^{2x}$.

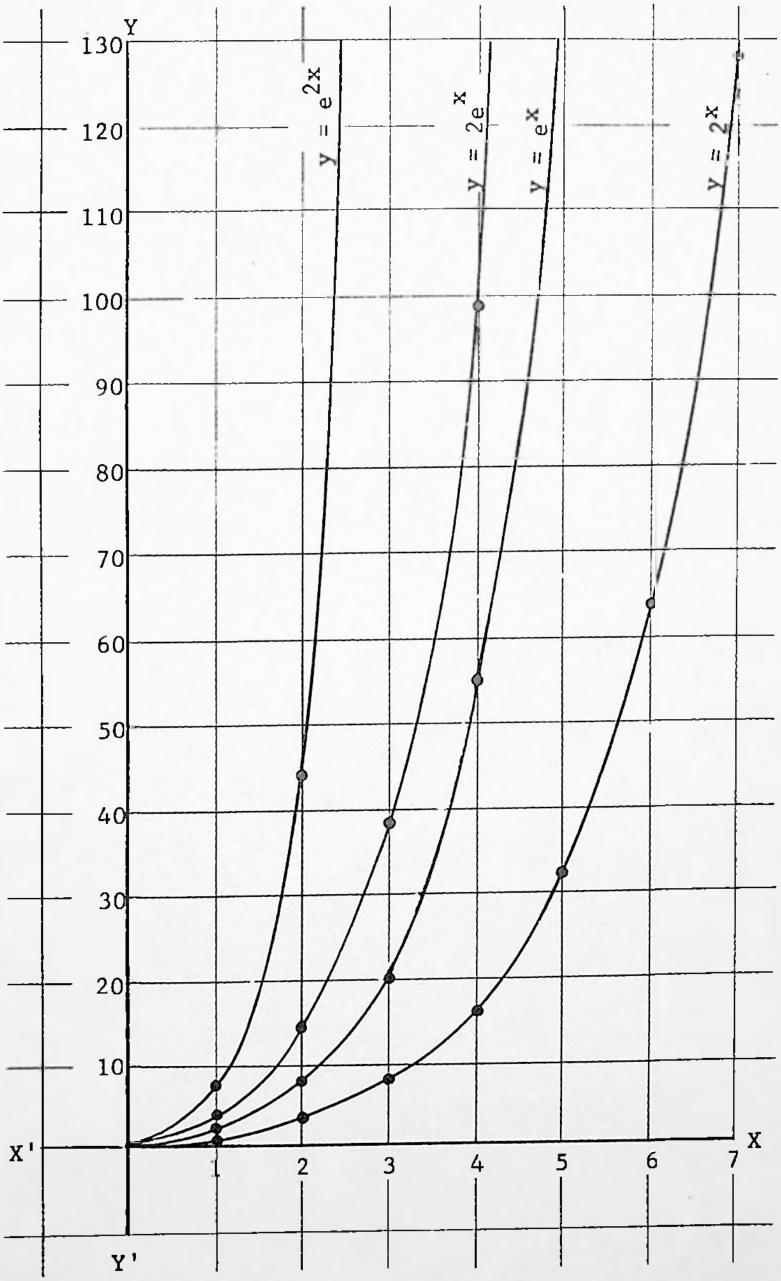


Figure 42. Graphs of exponential equations.

TABLE VI

TABLE OF VALUES TO ACCOMPANY FIGURE 42

$y = 2^x$		$y = e^x$		$y = 2e^x$		$y = e^{2x}$	
x	y	x	y	x	y	x	y
0	1	0	1	0	1.	0	1.
1	2	1	2.7	1	5.4	1	7.4
2	4	2	7.4	2	14.8	2	54.6
3	8	3	20.1	3	40.2		
4	16	4	54.6	4	109.2		
5	32	5	148.0				
6	64						
7	128						

(4) The solutions of some exponential equations can be determined by inspection. For example, in the equation $x^3 = 27$ it is necessary to find some value of x that, when cubed, will equal 27. Or, in $2^x = 16$ it is necessary to find the power to which 2 must be raised to obtain 16. In general, taking the logarithm of both sides of an exponential equation will result in a logarithmic equation that can be solved by the usual methods.

Example 1:

$$4^x = 256$$

$$x \log 4 = \log 256$$

$$x = \frac{\log 256}{\log 4} = \frac{2.4086}{.6021}$$

$$x = 4$$

Example 2:

$$y = 3e^2$$

$$\log y = \log 3 + 2 \log e \qquad (e = 2.71828)$$

$$= .4771 + 2(.4343)$$

$$= .4771 + .8686 \quad = 1.3457$$

$$y = 22.2$$

b. Logarithmic Functions.

(1) In previous lessons we made use of the table of logarithms. Using this table we can find the logarithm (in the base 10) of any positive number. In other words, this table represents a functional relation in which the independent variable can have any positive value

and the dependent variable is the logarithm of the independent variable. Letting x be the independent variable and y the dependent variable, we have the equation $y = \log_{10}x$. Although x is necessarily positive, y may be positive or negative. Some of these values are given in table VII.

TABLE VII

RELATED VALUES FOR $y = \log_{10}x$

x	y
.10	9.0000−10, or −1.000
.50	9.6990−10, or −.3010
1.00	0
3.00	.4771
5.0	.6990
7.0	.8451
10.0	1.0000

(2) Although logarithms can be computed in any base, the most commonly used bases are *ten* and *e*. The electronic technician will find frequent references to functions involving the natural logarithms (base e). The function is expressed by $y = \log_e x$ and table VIII lists the coordinates of several of the points that lie on the curve.

TABLE VIII

RELATED VALUES FOR $y = \log_e x$

x	y
.10	7.697−10, or −2.303
.50	9.307−10, or −.693
1.00	0
3.00	1.099
5.0	1.609
7.0	1.950
10.0	2.303

(3) Graphs for the logarithmic functions are made in the usual manner. Using the values from tables VII and VIII, the curves can be determined as shown in figure 43.

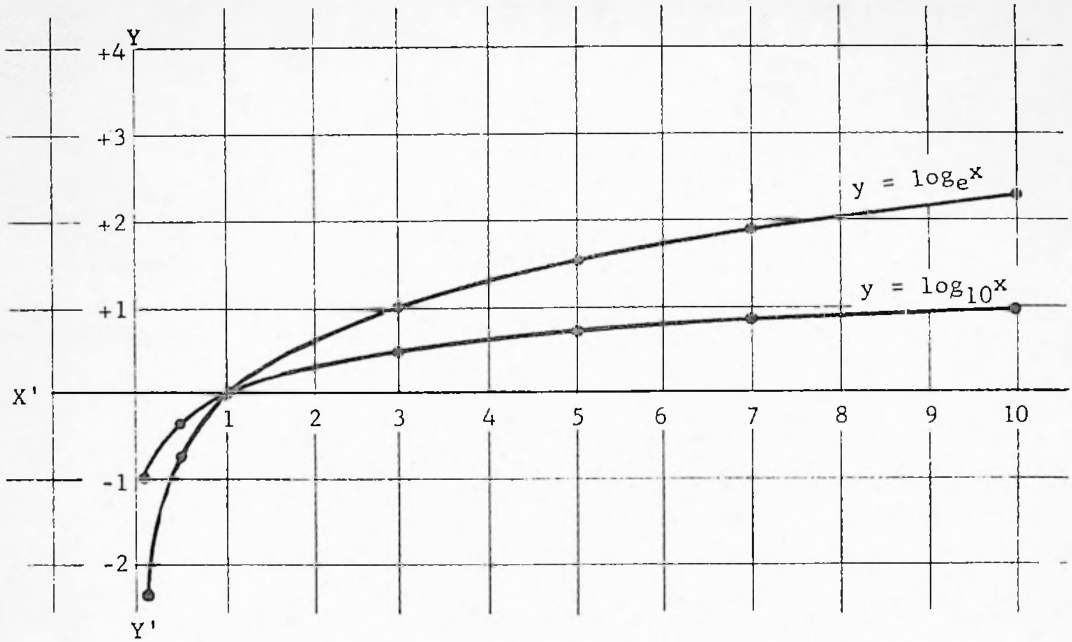


Figure 43. Graphs of $y = \log_e x$ and $y = \log_{10} x$.

c. Trigonometric Functions.

(1) In lesson 5 we learned that the trigonometric functions assume different values depending on the size of the angle. Thus, y = sin x, where x may be degrees or radians. Since electrical quantities are considered to have varying directions, the angle notation can be readily adapted to a study of electricity. Furthermore, the periodic nature (values repeating themselves at regular intervals) of alternating electricity permits us to say that the angle size is proportional to time. This is illustrated in table IX. A more general form of the sine function is y = a sin bx.

TABLE IX

RELATED VALUES FOR y = sin x

Degrees	x	0°	30°	60°	90°	180°	270°	360°	390°	etc
Microseconds		0	1	2	3	6	9	12	13	etc
	y	0	.5	.866	1	0	−1	0	.5	etc

(2) The graph of the function of y = sin x is shown in figure 44. Note that the curve begins to repeat itself at 12 microseconds (usec). Thus, one cycle of the curve is covered in 12 microseconds and every 12 microseconds thereafter. This illustrates the periodic nature of the sine function. The effect of the constants *a* and *b* in the equation y = a sin bx may be seen in figure 45. It is apparent that *a* affects the height or amplitude of the wave, while *b* affects the neighboring cycles or the frequency of the wave.

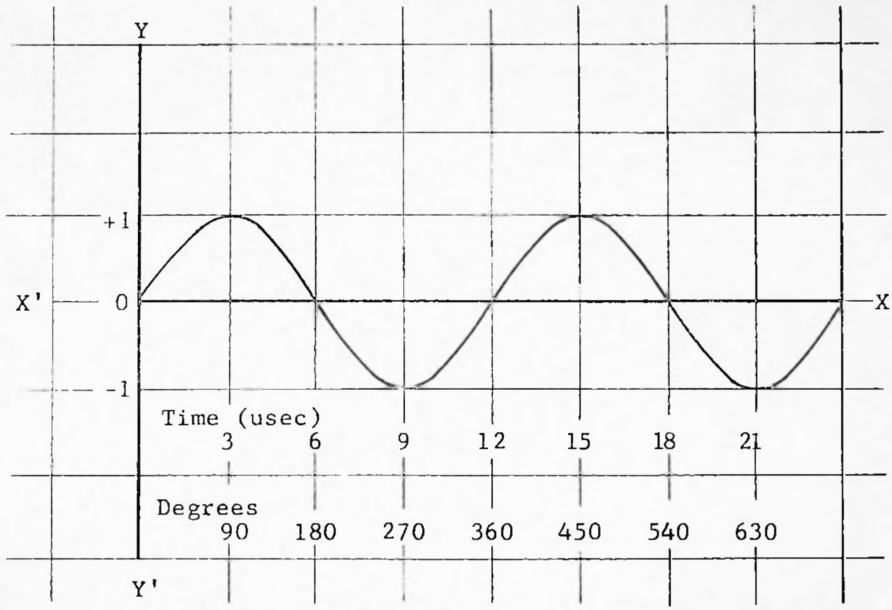


Figure 44. Graph of y = sin x.

(3) Taking y = cos x, let us assume values for x as was done in plotting y = sin x. The corresponding values of y are shown in table X. Note that the cosine curve is also periodic since the whole cycle (360°) is repeated as shown in figure 46.

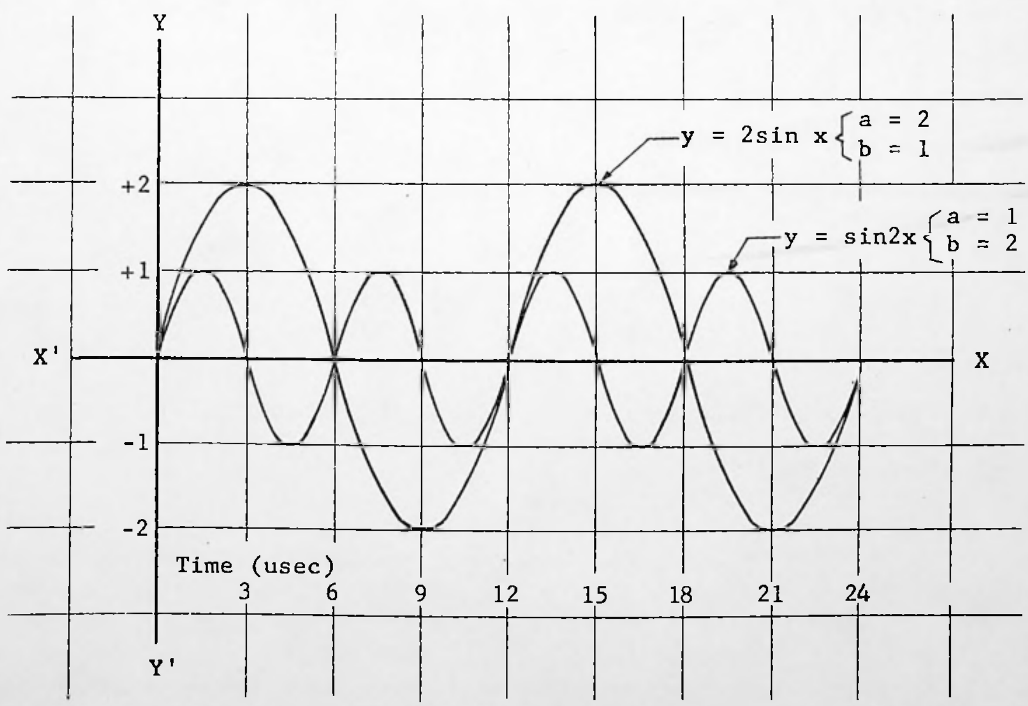


Figure 45. Graph of y = a sin bx.

TABLE X

RELATED VALUES FOR y = cos x

Degrees	x	0°	30°	60°	90°	180°	270°	360°	390°	etc
Microseconds		0	1	2	3	6	9	12	13	etc
	y	1	.866	.5	0	−1	0	1	.866	etc

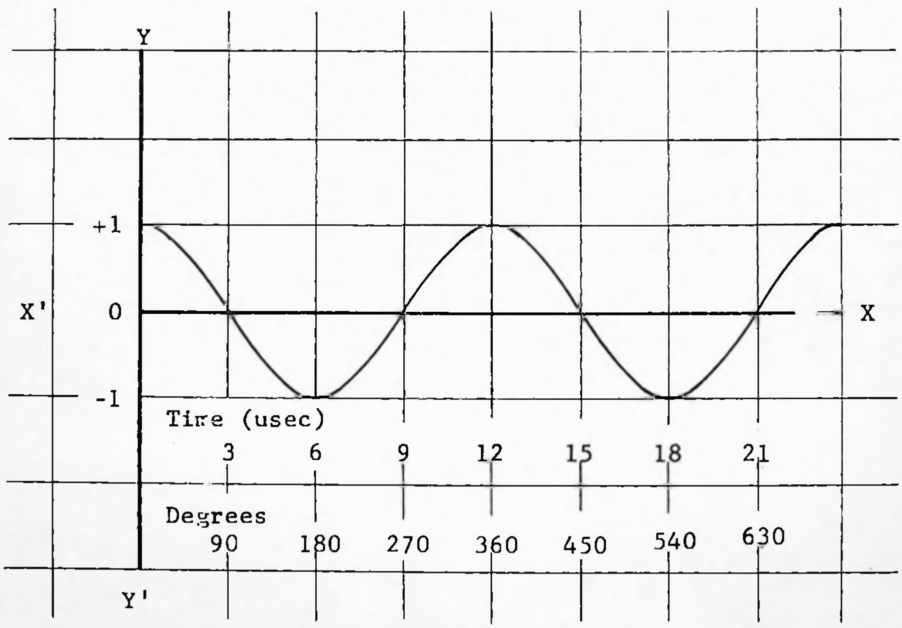


Figure 46. Graph of y = cos x.

12. PERIODICITY

a. From the graphs plotted in the preceding sections, it is apparent that each trigonometric function repeats itself exactly in the same order at regular intervals. A function that repeats itself periodically is called a *periodic* or *cyclic* function. Due to the fact that many natural phenomena are periodic, the sine and cosine curves lend themselves well to graphic representation and mathematical analysis of the recurrent motions. For example, the rise and fall of tides, the vibrations of a pendulum, sound waves, water waves, and alternating current are all happenings that can be represented and analyzed by the use of these curves. The tangent, cotangent, secant and cosecant curves are not used to represent recurrent events because, although these curves are periodic, they are undefined for certain angles.

b. In order to relate the size of an angle to time, it is necessary to introduce the term *angular velocity,* which is expressed as the angle generated by a rotating line (r) in a given period of time (fig 47).

Thus, angular velocity is measured in *degrees per second* or *radians per second.* Angular velocity may also be expressed in revolutions per second (rps). For example, if f is the number of revolutions per second of the rotating line in figure 47, then $2\pi f$ is the number of radians generated per second. The angular velocity in radians per second is denoted by ω (Greek letter *omega*). Thus, if the line is rotating f revolutions per second, $\omega = 2\pi f$ radians per second.

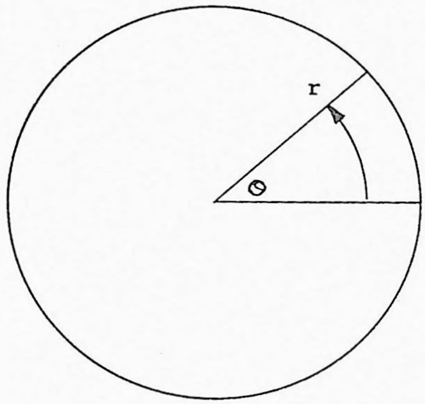


Figure 47. Determining angular velocity of a rotating line.

c. If the armature of a generator is rotating at 1,800 revolutions per minute (rpm), which is 30 revolutions per second, it has an angular velocity of $\omega = 2\pi f = 2 \times \pi \times 30 = 188.4$ radians per second. The total angle σ generated by the rotating line of figure 47 in t seconds at an angular velocity of ω radians per second is $\sigma = \omega t$. Thus the angle generated by the armature in 0.01 second is

$$\sigma = \omega t = 188.4 \times 0.01 = 1.884 \text{ radians}$$

$$\sigma = 1.884 \times 57.30 = 108^\circ.$$

Remember that if the angle is greater than 360°, the trigonometric function of the angle will be the same as that of the angle found by subtracting an integral multiple of 360°.

Thus, $\sin 420^\circ = \sin(420^\circ - 360^\circ) = \sin 60^\circ$

$\cos 790^\circ = \cos(790^\circ - 2 \times 360^\circ) = \cos 70^\circ.$

13. SUMMARY

Since most electrical quantities can be measured, an understanding of electricity and electronics requires the ability to visualize mathematical relationships. We may be required to know how a change in voltage will change the current or power in a circuit, or how a change in the time will affect the charge on a capacitor or voltage in an ac circuit. A formula should be thought of as a relationship between variable quantities rather than a means of obtaining a particular answer. In fact, functional relationships are so important in electricity and radio that too much emphasis cannot be placed on the subject.

EXERCISES

In each of the following exercises, select the ONE answer that BEST completes the statement or answers the question. Indicate your solution on the answer sheet by filling in with pencil the appropriate lettered block.

All exercises of this lesson are of equal weight. The total weight is 100.

121. The distance (s) that a body falls is proportional to the square of the time (t) that it takes to fall. Thus, $s = \frac{1}{2} gt^2$, where g is the gravity constant. The formula that expresses the time as a function of the distance traveled is

a. $t = \frac{2s}{g^2}$.
b. $t = \frac{2s^2}{g}$.
c. $t = 4\frac{s}{g}$.
d. $t = \sqrt{\frac{2s}{g}}$.

122. The following table gives the values of I and related values of P. After drawing the graph of this relationship, you can determine that when I has values of 0, 55, and 70, the corresponding values of P will be

I	10	20	30	40	50	60
P	0.1	0.5	1.1	1.9	3.0	4.3

a. 0, 2.4, and 5.0.
b. 0, 3.6, and 5.9.
c. 1, 3.6, and 5.0.
d. 1, 2.4, and 5.9.

123. It has been found that the points $(-2, 5)$ and $(3, -4)$ lie on the same curve. These points are found to be in quadrants

a. I and III.
b. II and III.
c. II and IV.
d. III and IV.

124. The equation $y = 2x - 4$ is represented on a graph as a straight line. Therefore, the line will pass through quadrants

a. I, II, III and IV.
b. I, II, and III.
c. I, III, and IV.
d. III and IV only.

125. The graph of the equation $y = -4$ is a straight line. Through which of the quadrants will this line pass?

a. III only
b. II and III only
c. III and IV only
d. all four quadrants

126. You are to find the point of intersection of two lines on a graph. Studying figure 48, you see that the graph of equation $2x + y = 12$ is already drawn. After graphing equation $x - 3y = -22$, you find the point of intersection to be

a. (0, 12).
b. (2, 8).
c. (4, 4).
d. (8, 10).

127. The intensity of illumination (I) which an object receives varies directly as the intensity of the source of light (s) and inversely as the square of the distance (d) from the source. The equation representing this statement is

a. $I = ksd^2$.
b. $I = \frac{ks}{d^2}$.
c. $I = \frac{k\sqrt{s}}{d}$.
d. $I = \frac{ks}{\sqrt{d}}$.

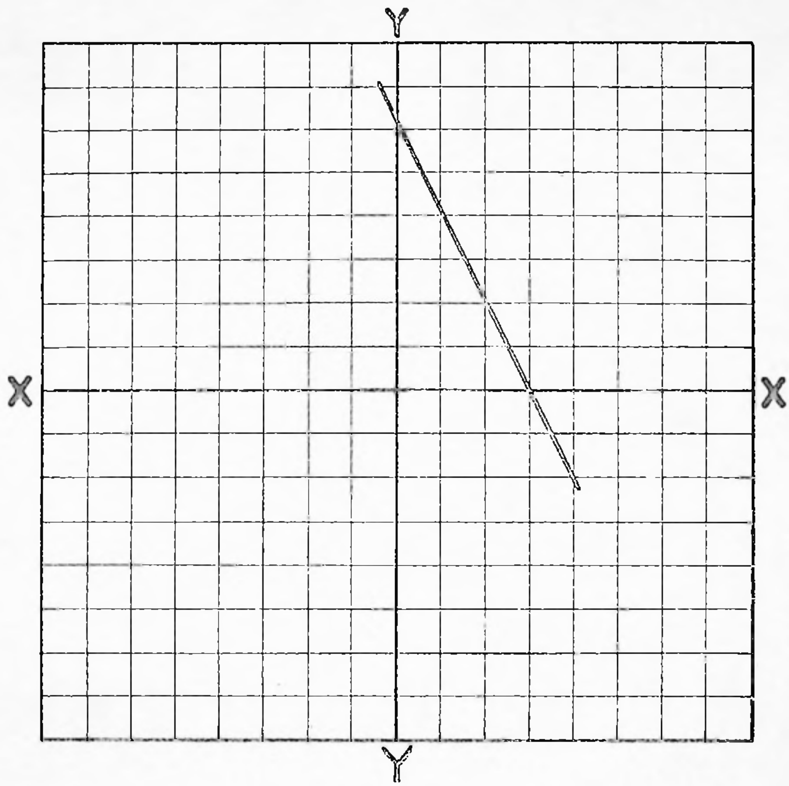


Figure 48. Graph of $2x + y = 12$.

128. In most problems involving related quantities, the constant of variation must be evaluated. For example, the voltage (E) in a circuit is directly proportional to the current (I). If the current is found to be 40×10^{-3} amperes and the voltage 6 volts, the constant of variation (resistance in this case) is

a. 15.
b. 24.
c. 150.
d. 240.

129. Two cable ships left port to lay a cable connection. Cable ship A left at 0600 hours at a speed of 10 knots (a knot is one nautical mile per hour). At 0900 hours, cable ship B left on the same course at a speed of 15 knots. The elapsed time and distance traveled when ship B overtakes ship A are

a. 1400 hours, 85 miles out.
b. 1400 hours, 90 miles out.
c. 1500 hours, 90 miles out.
d. 1600 hours, 100 miles out.

HINT: Prepare a graph plotting the distance in nautical miles traveled by each ship against the elapsed time. The desired information will be determined by the point of intersection of the two graphs.

130. If the operating frequency f (in mc) of a radio transmitter is known, the length L (in feet) of a quarter-wave antenna may be computed from the equation $L = \frac{234}{f}$. A quick method of determining the correct antenna length for a given frequency may be devised by plotting the graph of the equation. If the graph is drawn using values up to 50 mc for frequency, you can determine that for a given change in frequency, the greatest change in antenna length will occur

a. below 5 mc.
b. between 5 and 15 mc.
c. between 15 and 25 mc.
d. above 25 mc.

131. In an electrical circuit where the voltage is held at a constant 12 volts, the current I and resistance R may be expressed by the equation $IR = 12$. The shape of the graph of this equation is a

a. circle. *b.* parabola. *c.* hyperbola. *d.* straight line.

132. The resistance R of a wire varies inversely as the cross-sectional area A. If the constant of variation k is 0.01 and only positive values of area are considered, in which of the quadrants will the graph of the equation lie?

a. I only *b.* III only *c.* I and II *d.* I and III

133. Semilogarithmic graph paper (fig 49) is often used to show the relationship between the power level (db) and the ratio between output and input power. Use this figure to plot the gain of an amplifier (in db) against the power ratio (P2/P1), using the formula $db = 10 \log P2/P1$. The curve of this equation on semilogarithmic paper has the form of a

a. straight line.
b. hyperbola.
c. parabola.
d. logarithmic curve.

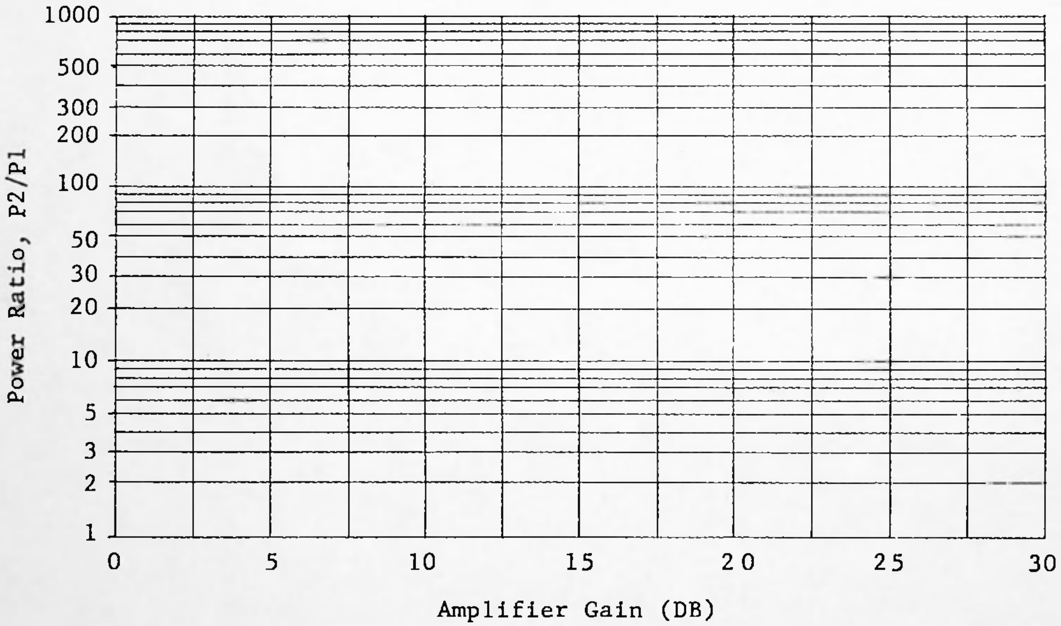


Figure 49. Semilogarithmic graph paper.

SITUATION

At any instant, the voltage across a capacitor C discharging through a resistor R may be expressed by an exponential equation. If $E_B = 100$ volts, $R = 50{,}000$ ohms, and $C = 0.2 \times 10^{-6}$ farad, the equation is reduced to $E_c = 100\,e^{-100t}$. Using the pairs of numbers in the following table, plot on figure 50 the curve showing the relationship between time t and voltage E_c. The missing value of E_c at $t = 0.01$ may be computed from the formula by using logarithms.

t (sec)	0	.005	.01	.02	.03	.04	.05
E_c (volts)	100	60.6		13.5	5.0	1.8	0.6

Exercises 134 and 135 are based on the above situation.

134. How much time is required for the capacitor voltage to drop to one half of its original voltage?

a. 0.007 second
b. 0.009 second
c. 0.011 second
d. 0.014 second

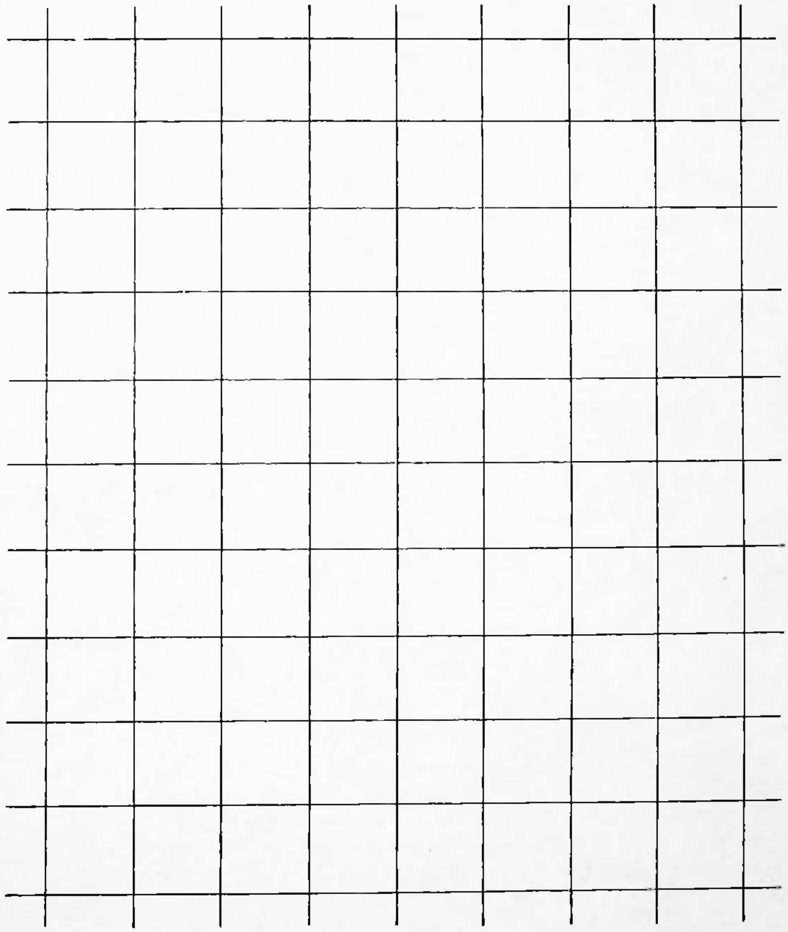


Figure 50. Graph paper for exercises 134 and 135.

135. The change in voltage between t = 0.015 second and t = 0.025 second is approximately

a. 4 volts.
b. 14 volts.
c. 27 volts.
d. 38 volts.

136. The graph of y = sin x + cos x can be plotted by adding the values of sin x and cos x for various values of x (use fig 51). From this graph it can be determined that the curve is increasing when the values of x are approximately

a. 20° and 170°.
b. 40° and 270°.
c. 60° and 315°.
d. 80° and 200°.

137. The power in an ac circuit may be computed from the equation P = EI cos A where A is the phase angle between the voltage and current. Assuming that E = 220 volts and I = 2 amps,

draw a graph (use fig 51) showing power as a function of the phase angle (A). From the graph it may be determined that the power has a maximum value of

a. 220 watts at 270°.
b. 220 watts at 360°.
c. 440 watts at 270°.
d. 440 watts at 360°.

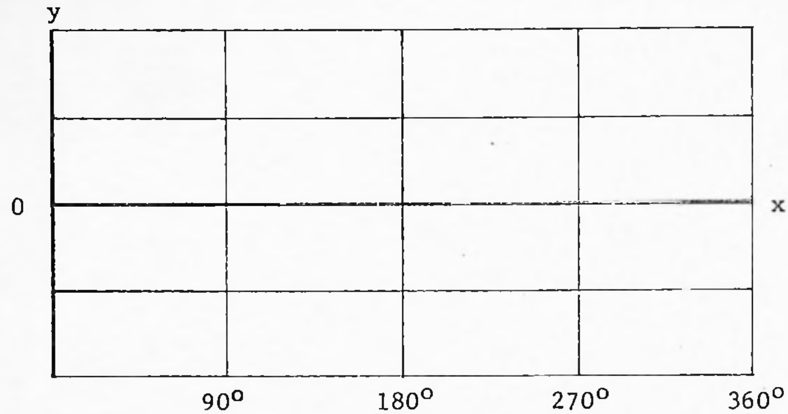


Figure 51. Graph paper for exercises 136 and 137.

138. Assume that an FM transmitter is being modulated with a sine wave signal so that its frequency deviation can be expressed by the equation $D = 524 \cos \omega t$. If the frequency (f) is 1,000 cps, the deviation at 1.25×10^{-4} sec is

a. 262 cps.
b. 370 cps.
c. 454 cps.
d. 524 cps.

HINT: In this problem the 1,000 cps is equivalent to rev per sec for purposes of determining ω.

139. The formula for finding the illumination on a surface that is NOT perpendicular to the rays of a light source is $E = \frac{I \cos \theta}{d^2}$. I is the intensity of the light source (in candles), d is the distance (in feet) from the light source, E is the illumination on the surface that is not perpendicular to the rays of the light source, and θ is the angle between the perpendicular line to the surface and the ray of light. To find the distance (in feet) from the light source we have

a. $d = \sqrt{\frac{E}{I \cos \theta}}$.
b. $d = \sqrt{\frac{I \cos \theta}{E}}$.
c. $d = \frac{I \cos \theta}{E}$.
d. $d = \frac{E\theta}{I \cos}$.

140. Assume that the armature of a generator is turning at 60 revolutions per second. The angle through which the armature passes in $\frac{1}{125}$ second is

a. 3 degrees.
b. 92 degrees.
c. 172 degrees.
d. 300 degrees.

A REMINDER

You will need the text materials to complete the examination of this subcourse. Therefore—keep all texts until the School notifies you that you have **successfully completed** the subcourse.